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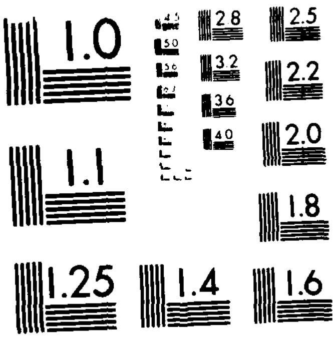

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FIRST ANNUAL REPORT

on

STOCHASTIC NON-LINEAR FLUTTER OF AEROELASTIC STRUCTURES

October 21, 1985

SEAL OF TEXAS TECH UNIVERSITY

Prepared for

Air Force Office of Scientific Research
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Prepared by
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TEXAS TECH UNIVERSITY

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The linear and non-linear random modal interactions of a two degree-of-freedom aeroelastic structure are examined by using the Fokker-Planck equation approach. A general differential equation describing the evolution of the response moments is derived for any moment order. For the case of linear modal interaction this differential equation is found to constitute a closed set of moment equations. The stationary response is determined for various system parameters. It is found that the linear interaction results in a suppression of one mode when the uncoupled frequencies of the structure are close to each other. For the case of non-linear modal (known as autoparametric) interaction the differential equation of the response moments forms an infinite coupled set of equations which are closed via two closure schemes. These are the Gaussian and non-Gaussian closure schemes. The Gaussian closure scheme requires 14 coupled differential equations in the first and second order moments, while the non-Gaussian closure leads to 69 differential

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1. ABSTRACT

The linear and non-linear random modal interactions of a two degree-of-freedom aeroelastic structure are examined by using the Fokker-Planck equation approach. A general differential equation describing the evolution of the response moments is derived for any moment order. For the case of linear modal interaction this differential equation is found to constitute a closed set of moment equations. The stationary response is determined for various system parameters. It is found that the linear interaction results in a suppression of one mode when the uncoupled frequencies of the structure are close to each other. For the case of non-linear modal (known as autoparametric) interaction the differential equation of the response moments forms an infinite coupled set of equations which are closed via two closure schemes. These are the Gaussian and non-Gaussian closure schemes. The Gaussian closure scheme requires 14 coupled differential equations in the first and second order moments, while the non-Gaussian closure leads to 69 differential equations in the first four orders of response moments. The two sets are solved by numerical integration. Both solutions exhibit an energy exchange between the two modes in the neighborhood of the internal resonance condition. The Gaussian closure solution gives a quasi-stationary response in the form of fluctuations between two limits. However, the non-Gaussian closure solution results in a strict stationary response. The influence of random time fluctuations in the system damping and stiffness coefficients is also examined. It is found that the damping variation has very small effect on the response characteristics, while the stiffness variation shows a pronounced effect on the response mean squares for both solutions.

NOMENCLATURE

A_i	coefficients of the Markov vector equations (III.7)
a_i	coefficients of equations (III.7)
B_i	coefficients of the Markov vector equations (III.7)
$B_i(\tau)$	Brownian motion
b_i	coefficients of equations (III.7)
C_{ii}	generalized damping coefficient (i = 1, 2)
2D	spectral density of the support motion acceleration
$2D_{ci}$	spectral density of the random damping fluctuation C_i
$2D_{ki}$	spectral density of the random stiffness fluctuation K_i
d_i	coefficients of moments matrix (equation II.17)
E[ ]	expectation
E_i	Young's modulus, or coefficients of equations (II.10)
e_i	coefficients of moment equations (II.17)
f_i	coefficients of moments matrix (equation II.17)
$f_i(\underset{\sim}{X},\tau)$	elements of a column matrix equation (IV.2)
G_i	coefficients of equations (II.10)
$G_{ij}(\underset{\sim}{X},\tau)$	elements of the parametric excitation matrix equation (IV.2)
I_i	area moment of inertia of beam's cross section (i = 1, 2)
K_{ii}	generalized stiffness coefficient (i = 1, 2)
ℓ_i	beam lengths (i = 1, 2)
ℓ_{ij}	elements of the matrix of parametric excitation
ℓ_{10}	coefficient of the non-homogeneous part of the excitation
m_i	tip masses (i = 1, 2)
m_{ij}	generalized mass (i = 1, 2; J = 1, 2)
$m_{ijk\ell}$	joint moments of response coordinates of order $i+j+k+\ell$

$p(\underset{\sim}{X},\tau)$	joint (transition) probability density function of the response coordinates
q_i	generalized coordinates
q_i^*	axial shortening of the beams ($i = 1, 2$)
q_1^o	root-mean-square of the horizontal beam where the vertical beam is locked
$[R]$	modal matrix
r	frequency ratio
t	time (seconds)
$W(\tau)$	white noise
X_i	state space coordinates
Y_i	non-dimensional normal coordinates
y_i	normal coordinates ($i = 1, 2$)
$\delta(\ )$	Dirac delta function
ε	non-linear coupling parameter
ζ_i	damping factor of normal coordinates
$\lambda_i[\ ]$	joint cumulant of order i
μ	mass parameter defined by equation (II.2)
ξ_{ci}	random variation in the damping coefficient C_i
ξ_{ki}	random variation in the stiffness coefficient K_i
$\ddot{\xi}(t)$	random support acceleration
σ_i^2	spectral density
τ	non-dimensional time $= \omega_1 t$
ϕ_i	elements of eigenvectors
$\psi_i(q_i,\dot{q}_i,\ddot{q}_i)$	non-linear terms in equations (I.1)
ω_i	normal mode frequencies
ω_{ii}	local beam frequencies

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2. RESEARCH OBJECTIVES

2.1 BACKGROUND

The dynamic behavior of aeroelastic structures is of main concern to aeronautical engineers who are involved in the design and reliability of aerospace structures. These structures are usually subjected to aerodynamic forces which interact with inertia and elastic forces. The interaction of these forces may give rise to a number of aeroelastic phenomena. For example, the classical flutter (known also as self-excited oscillation) can occur due to a linear interaction of these three forces. Classical flutter may also involve the coupling of two or more degrees of freedom. However, the linear mathematical modeling fails to predict a number of observed dynamic characteristics such as amplitude jump, limit cycles, parametric instability, internal resonance, and saturation phenomenon. These complex dynamic characteristics owe their origin to the inherent non-linearity of the structure.

The amplitude jump, limit cycles, and parametric instability are common features of non-linear single- and multi-degree-of-freedom systems. Parametric instability takes place when the external excitation appears as a coefficient in the homogeneous part of the differential equation of motion. It occurs when the excitation frequency is twice (or multiple) of the system natural frequency. Internal resonance and saturation phenomenon may occur only in non-linear dynamic systems with more than one degree-of-freedom. Internal resonance implies the existence of a linear relationship between the structure natural frequencies and results in a non-linear interaction of the normal modes in a form of energy exchange. Under external excitation, the mode which is directly excited exhibits in the

beginning, the same features of a single degree-of-freedom system response and all other modes remain dormant. As the excitation amplitude reaches a certain critical level, the other modes become unstable and the originally excited mode reaches an upper bound. In this case this mode is said to be saturated and the energy "spills over" into other modes. The non-linear modal interaction is referred in the literature as autoparametric interaction since one mode acts as a parametric excitation to another mode. The non-linearities in any structural component may arise from two main sources:

1. Geometric non-linearities due to large deformations such as large curvature, end shortening, and inertias due to the presence of concentrated or distributed masses.

2. Material properties which exhibit non-linear or multi-valued stress-strain relationships. The values of the material properties may also experience a certain degree of uncertainty due to material heterogeneity or random temperature fluctuations. Thus the stiffness and damping coefficients become random variables or random processes.

It is clear that the aeroelastician must include the inherent structural non-linearities in his predictive models in order to understand the origin of any unusual structure behavior under various types of aerodynamic loading. Under deterministic unsteady aerodynamic forces these phenomena can be predicted by one of the standard techniques of non-linear differential equations. However, aerospace structures are usually subjected to turbulent air flow, and the analyst is encountered with aerodynamic loads

which are random in nature. These loads vary in a highly irregular fashion and can be described in terms of statistical quantities such as means, mean square, autocorrelation functions, and spectral density functions. The dynamic analysis of non-linear aeroelastic structures under random loading is not a simple task, and it requires an advanced background of probabilistic theory and stochastic differential equations.

2.2 MAIN OBJECTIVES

In an effort to understand the dynamic behavior of non-linear aeroelastic structures under random excitations a research program consisting of analytical and experimental investigations is currently supported by a grant from the Air Force Office of Scientific Research (AFOSR). Two and three degree-of-freedom systems possessing internal resonance are considered. The experimental investigation will demonstrate the existence of non-linear phenomena and will provide guidelines for the validity of the theoretical analysis. Based on the original proposal (February 1983) and its amendment (July 1984) three main phases were outlined. These are:

Phase I: Investigation of the effects of structural non-linearities in the neighborhood of internal resonance conditions when the structure is subjected to random aerodynamic loading.

Phase II: Investigation of the effects of damping and stiffness uncertainties in the absence of internal resonance conditions.

Phase III: Investigation of the combined effects of damping and stiffness uncertainties in the presence of internal resonance conditions.

The experimental investigation will be carried out on two models emulating the analytical models. The models will be excited by a medium size electrodynamic shaker of 1200 lb maximum thrust through a GenRand random noise generator. The excitation will be Gaussian filtered wide band process whose band covers a frequency range greater than the normal mode frequencies considered in the analytical models. The excitation frequency band will be adjusted such that higher modes will not be excited.

The excitation and response processes will be measured and recorded simultaneously on a magnetic tape recorder. The mean squares and probability density of the response will be estimated for various values of internal resonance detuning. These results will be very valuable in demonstrating how the normal modes are interacting under random excitation. In addition, the measured probability density will be inspected for its non-normality when the excitation is Gaussian.

2.3 SUMMARY OF MAIN RESULTS

The linear and non-linear random modal interactions of a two degree-of-freedom aeroelastic structure are examined by using the Fokker-Planck equation approach. A general differential equation describing the evolution of the response moments is derived for any moment order. For the case of linear modal interaction this differential equation is found to constitute a closed set of moment equations. The stationary response is determined for various system parameters. It is found that the linear interaction

results in a suppression of one mode when the uncoupled frequencies of the structure are close to each other. For the case of non-linear modal (known as autoparametric) interaction the differential equation of the response moments forms an infinite coupled set of equations which are closed via two closure schemes. These are the Gaussian and non-Gaussian closure schemes. The Gaussian closure scheme requires 14 coupled differential equations in the first and second order moments, while the non-Gaussian closure leads to 69 differential equations in the first four orders of response moments. The two sets are solved by numerical integration. Both solutions exhibit an energy exchange between the two modes in the neighborhood of the internal resonance condition. The Gaussian closure solution gives a quasi-stationary response in the form of fluctuations between two limits. However, the non-Gaussian closure solution results in a strict stationary response. The influence of random time fluctuations in the system damping and stiffness coefficients is also examined. It is found that the damping variation has very small effect on the response characteristics, while the stiffness variation shows a pronounced effect on the response mean squares for both solutions.

3. STATUS OF THE RESEARCH

CHAPTER I

BASIC MODEL AND EQUATIONS OF MOTION

I.1 Two-Degree-of-Freedom System

Figure I.1 shows a schematic diagram of a two-degree-of-freedom aeroelastic structural model which represents an aircraft wing with external store. The model consists of two coupled beams of stiffnesses, k_1 and k_2, and two tip masses, m_1 and m_2. When the horizontal beam is subjected to a random motion $\xi(t)$, the two beams will move as shown in Fig. I.1 with tip deflections q_1 and q_2, respectively. By applying Lagrange's equation, and including the effects of the axial shortening motions, q_1^*, and q_2^* in the kinetic energy expression, the equations of motion are given in terms of the generalized coordinates q_1 and q_2:

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \begin{bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

$$= - \ddot{\xi}(t) \begin{bmatrix} \ell_{11} & \ell_{12} \\ \ell_{21} & \ell_{22} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} - \ddot{\xi}(t) \begin{Bmatrix} \ell_{10} \\ 0 \end{Bmatrix}$$

$$- \begin{Bmatrix} \psi_1(q_1, q_2, \dot{q}_1, \dot{q}_2, \ddot{q}_1, \ddot{q}_2) \\ \psi_2(q_1, q_2, \dot{q}_1, \dot{q}_2, \ddot{q}_1, \ddot{q}_2) \end{Bmatrix} \tag{I.1}$$

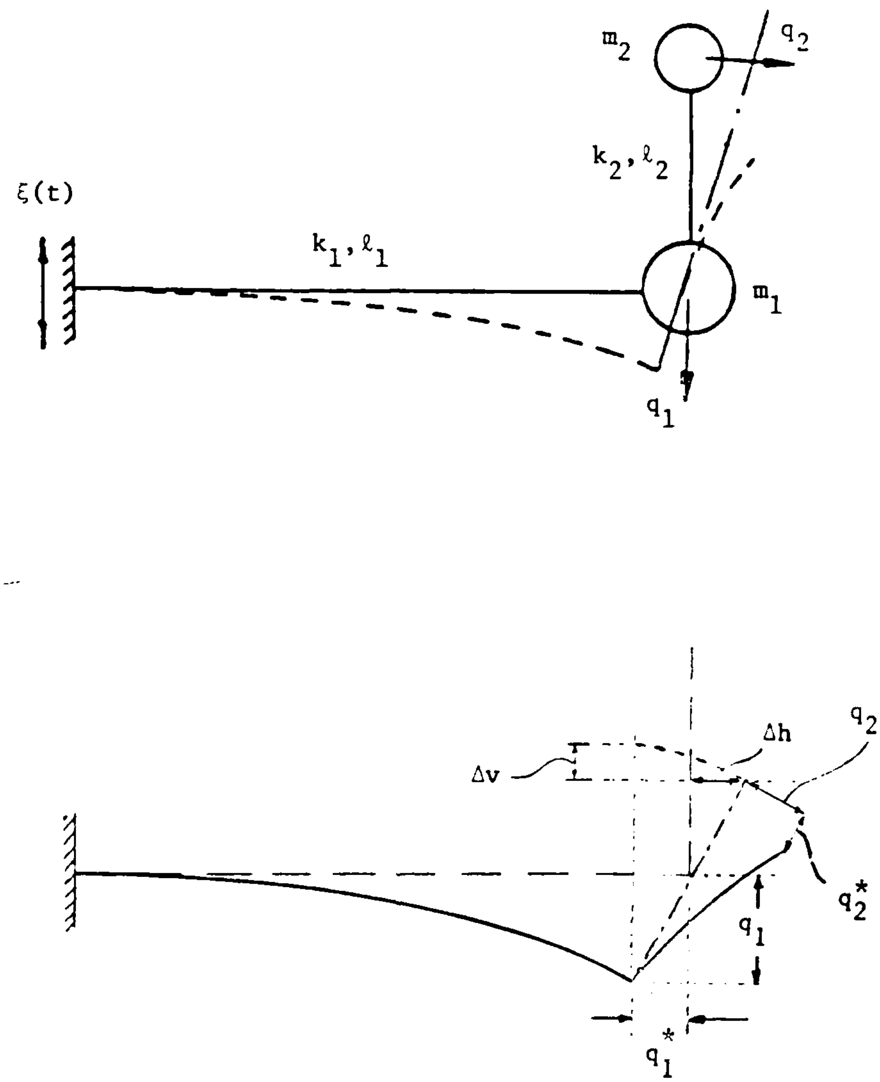


Fig. I.1 Schematic diagram of two coupled beams with end masses m_1 and m_2.

where $m_{11} = m_1 + m_2 \left[1 + 2.25 \left(\frac{\ell_2}{\ell_1}\right)^2\right]$

$$m_{12} = m_{21} = 1.5 \frac{\ell_2}{\ell_1} m_2, \quad m_{22} = m_2$$

$$\ell_{11} = 2.25\, m_2 \left(\frac{\ell_2}{\ell_1^2}\right) \qquad \ell_{10} = m_1 + m_2$$

$$\ell_{12} = \ell_{21} = 1.5 \frac{m_2}{\ell_2} \qquad \ell_{22} = 1.2 \frac{m_2}{\ell_2}$$

$$K_{ii} = \frac{3E_i I_i}{\ell_i^3} \qquad i = 1,2$$

$$\psi_1 = m_1 \frac{1.44}{\ell_1^2} \left(q_1^2 \ddot{q}_1 + q_1 \dot{q}_1^2 \right)$$

$$+ m_2 \left\{ 0.45 \frac{\ell_2}{\ell_1^2} \left(\dot{q}_1^2 + 2 q_1 \ddot{q}_1 \right) + \frac{1.2}{\ell_2} \left(q_2 \ddot{q}_2 + \dot{q}_2^2 \right) \right.$$

$$+ \frac{3}{\ell_1} \left(\dot{q}_1 \dot{q}_2 + \ddot{q}_1 q_2 + 0.1\, q_1 \ddot{q}_2 \right) + 1.6875 \frac{\ell_2}{\ell_1^3} q_1^2 \ddot{q}_2$$

$$- \frac{0.45}{\ell_1^2} \left(\ddot{q}_1 q_2^2 + 2 \dot{q}_1 q_2 \dot{q}_2 \right) + \frac{0.9}{\ell_1 \ell_2} \left(2 q_2 \dot{q}_2^2 + q_2^2 \ddot{q}_2 \right)$$

$$\left. + 5.0625 \frac{\ell_2^2}{\ell_1^4} \left(q_1 \dot{q}_1^2 + q_1^2 \ddot{q}_1 \right) + \frac{1.44}{\ell_1^2} \left(q_1 \dot{q}_1^2 + q_1^2 \ddot{q}_1 \right) \right\}$$

$$\psi_2 = m_2 \{\frac{0.3}{\ell_1} q_1\ddot{q}_1 - \frac{1.2}{\ell_1} \dot{q}_1^2 + \frac{1.2}{\ell_2} \ddot{q}_1 q_2$$

$$+ 1.6875 \frac{\ell_2}{\ell_1^3} (2q_1\dot{q}_1^2 + q_1^2\ddot{q}_1) + \frac{0.9}{\ell_1\ell_2} \ddot{q}_1 q_2^2$$

$$+ \frac{1.44}{\ell_2^2} (q_2\dot{q}_2^2 + q_2^2\ddot{q}_2) + \frac{0.45}{\ell_1^2} \dot{q}_1^2 q_2\}$$

Non-linearities up to cubic order are retained in ψ_1 and ψ_2. It is seen that the equations of motion contain linear dynamic coupling since $m_{ij} \neq 0$, $i \neq j$. In addition, the support motion acceleration $\ddot{\xi}(t)$ enters the equations of motion as a non-homogeneous forced excitation $\ell_{10}\ddot{\xi}(t)$ and as a parametric excitation as given by the terms $\ell_{ij}\ddot{\xi}(t)q_i$. The functions ψ_1 and ψ_2 include all nonlinear inertia terms. These can be subdivided into two groups. The first group represents non-linear inertia of the same mode such as $q_1^2\ddot{q}_1$ in the first equation. The second group represents auto-parametric coupling (or non-linear interaction) such as $q_1\ddot{q}_2$ in the first equation. Here the acceleration $\ddot{q}_2$ (which is an implicit function of time) acts as a parametric excitation to q_1 motion.

In deriving equations (I.1) it was assumed that both damping coefficients C_{ii} and beam stiffness K_{ii} are constant coefficients. In Chapter 4 these coefficients will be subject to random fluctuations.

In Chapter 2, the linear modal interaction under random excitation will be examined by dropping ψ_1 and ψ_2. In Chapter 3, the non-linear modal interaction will be investigated when the structure is tuned to the internal resonance condition $\omega_2 = 2\omega_1$ (where ω_1 and ω_2 are two eigenvalues of the structure). Chapter 4 will treat the influence of random fluctuations of the system parameters.

CHAPTER II

LINEAR ANALYSIS

II.1 Introduction

The normal mode interaction of the two-degree-of-freedom system shown in Fig. I.1 will be examined within the framework of the linear theory of random vibration. In other words, the system response represented by equations (I.1) will be analyzed after setting the non-linear functions ψ_1 and ψ_2 to zero. The normal mode frequencies and mode shapes will be obtained first, and then the equations of motion will be transformed into normal coordinates. These in turn will be written in the Itô type stochastic differential equations [1,2].

The differential equations of the response statistical moments will be derived by using the Fokker-Planck equation [1]. The Fokker-Planck equation is a partial differential equation which describes the evolution of the system response probability density with respect to time and response coordinates. In view of the linearity of the system equations of motion, the moment equations will form a closed set which can be solved numerically by one of the standard techniques. The mean square of the response moment will be examined for a wide range of system parameters. The influence of random parametric coefficients will also be examined.

II.2 Normal Mode Analysis

By dropping the non-linear function ψ_1 and ψ_2 from the equations (I.1), the system becomes linear with random coefficients:

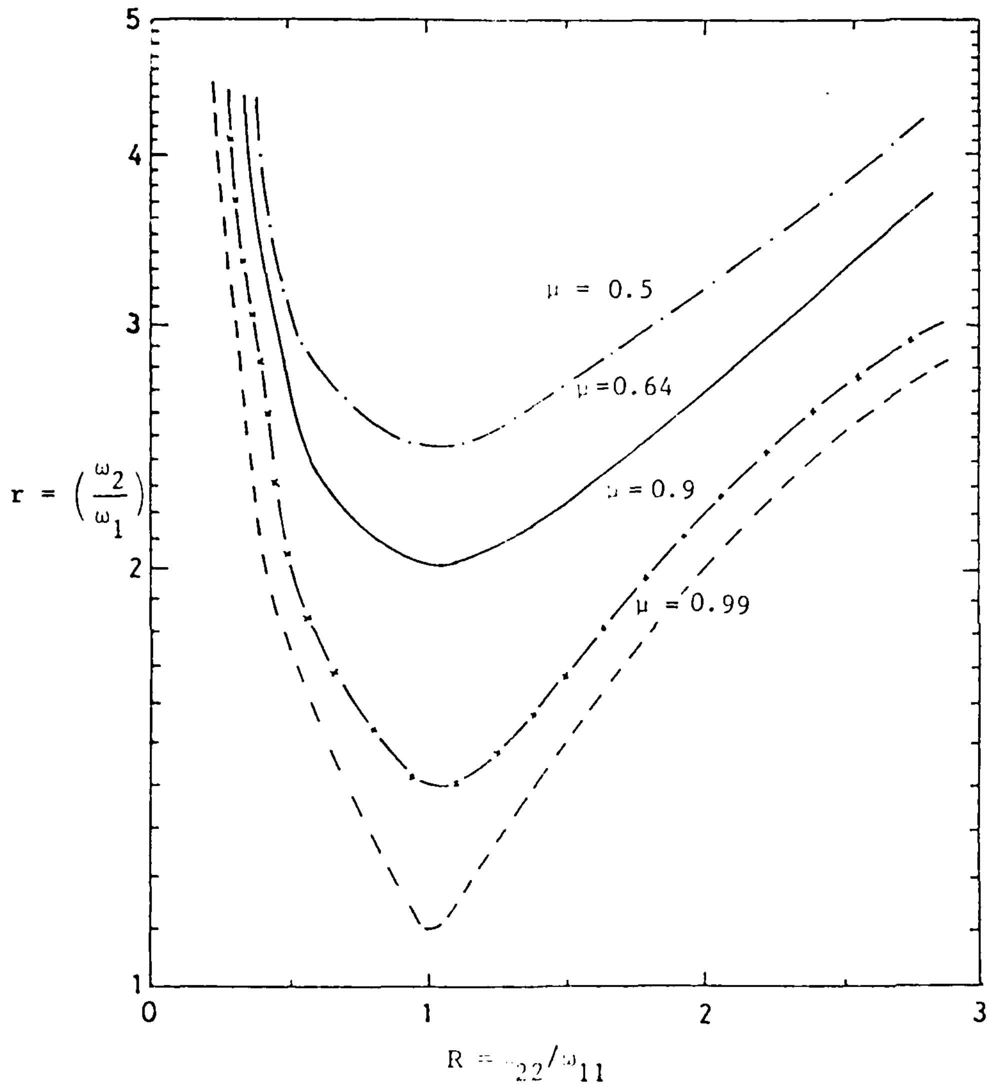


Fig. II.1 Dependence of normal mode frequency ratio r on the uncoupled natural frequency ratio R of the two beams for various values of mass parameter μ.

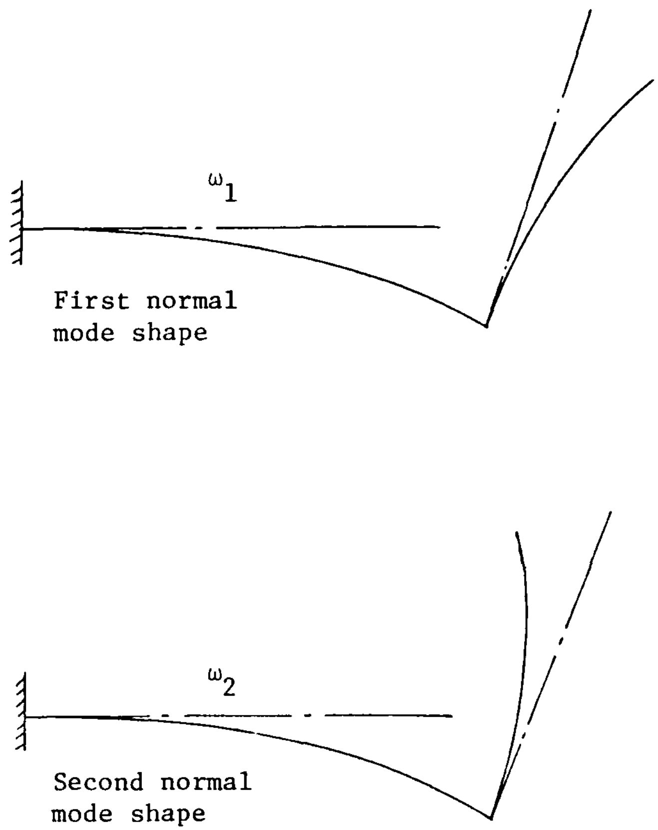


Fig. II.2 First two normal mode shapes.

$$\begin{bmatrix} m_{11} & m_{21} \\ m_{12} & m_{22} \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \begin{bmatrix} k_{11} & 0 \\ 0 & k_{22} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

$$= - \ddot{\xi}(t) \begin{Bmatrix} \ell_{10} \\ 0 \end{Bmatrix} - \ddot{\xi}(t) \begin{bmatrix} \ell_{11} & \ell_{12} \\ \ell_{21} & \ell_{22} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \tag{II.1}$$

The following notations are introduced:

$$\omega_{ii}^2 = \frac{k_{ii}}{m_{ii}} \quad = \text{local beam frequency}$$

$$\mu = 1 - \frac{m_{12}}{m_{11}} \frac{m_{21}}{m_{22}} = \text{mass parameter} \tag{II.2}$$

where $i = 1, 2$

The normal mode frequencies ω_1 and ω_2 of the system can be obtained by considering only the homogeneous conservative part of equations (II.1). In terms of the mass parameter μ and the local frequencies ω_{11} and ω_{22} the natural frequencies are obtained by the expression

$$\omega_{1,2}^2 = \frac{1}{2\mu} \left\{ \omega_{11}^2 + \omega_{22}^2 \mp \left((\omega_{11}^2 + \omega_{22}^2)^2 - 4\mu\omega_{11}^2\omega_{22}^2 \right)^{1/2} \right\} \tag{II.3}$$

Figure II.1 shows the dependence of ω_2/ω_1 on ω_{22}/ω_{11} for various values of the mass parameter μ.

Equations (II.1) can be transformed into normal coordinates y_i via the coordinate transformation

$$\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{bmatrix} R \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} \tag{II.4}$$

where $[R] = \begin{bmatrix} 1 & 1 \\ \phi_1 & \phi_2 \end{bmatrix}$ is the modal matrix,

and $\begin{Bmatrix} 1 \\ \phi_1 \end{Bmatrix}$, $\begin{Bmatrix} 1 \\ \phi_2 \end{Bmatrix}$ represent the eigenvectors corresponding to ω_1 and ω_2, respectively.

ϕ_1 and ϕ_2 are given by the expressions:

$$\phi_1 = \frac{\omega_{11}^2 - \omega_1^2}{\omega_1^2 \cdot \frac{m_{12}}{m_{11}}} \quad \text{or} \quad \frac{\omega_1^2 \cdot \frac{m_{21}}{m_{22}}}{\omega_{22}^2 - \frac{m_{22}}{m_{11}} \cdot \omega_1^2} \tag{II.5}$$

$$\phi_2 = \frac{\omega_{11}^2 - \omega_2^2}{\omega_2^2 \cdot \frac{m_{12}}{m_{11}}} \quad \text{or} \quad \frac{\omega_2^2 \cdot \frac{m_{21}}{m_{22}}}{\omega_{22}^2 - \frac{m_{22}}{m_{11}} \cdot \omega_2^2}$$

Figure II.2 demonstrates the normal mode shapes for mass parameter $\mu = 0.64$, and $\omega_{22} = \omega_{11}$.

The following non-dimensional parameters are introduced

$$\tau = \omega_1 t \quad , \quad Y_1 = \frac{y_1}{q_1^o} \quad , \quad Y_2 = \frac{y_2}{q_1^o}$$
$$\varepsilon = \frac{q_1^o}{\ell_1} \quad , \quad r = \frac{\omega_2}{\omega_1} \qquad \text{(II.6)}$$

where q_1^o is the root mean square response of the system when it is reduced into a single degree-of-freedom represented by the horizontal beam with end mass $(m_1 + m_2)$. Thus, $E[q_1^{o2}]$[1] is the mean square response of the single-degree-of-freedom system.

$$\ddot{q}_1^o + 2\zeta_{11}\omega_{11}\dot{q}_1^o + \omega_{11}^2 q_1^o = - W(t) \qquad \text{(II.7)}$$

The random excitation acceleration $\ddot{\xi}(t)$ has been replaced by the zero mean Gaussian white noise process W(t) whose autocorrelation function is defined by the relation

$$R[\Delta t] = E[W(t)W(t + \Delta t)] = 2D\,\delta(\Delta t) \qquad \text{(II.8)}$$

where 2D is the spectral density of the random process W(t) and $\delta(\;)$ is the Dirac delta function.

The mean-square response of system (II.7) is given by the well-known solution [3]:

$$E[q_1^{o2}] = \frac{D}{2\zeta_{11}\,\omega_{11}^3}$$

Thus, we may select q_1^o to be the root mean square of $E[q_1^{o2}]$; i.e.,

$$q_1^o = \frac{D}{2\zeta_{11}\,\omega_{11}^3} \qquad \text{(II.9)}$$

---

[1] E[ ] denotes expectation

The equations of motion in terms of the non-dimensional normal coordinates Y_i are

$$
\begin{aligned}
Y_1'' + 2\zeta_1 Y_1' + Y_1 &= \left[E_1 + \varepsilon(E_2 Y_1 + E_3 Y_2)\right] W(\tau) \\
Y_2'' + 2r\zeta_2 Y_2' + r^2 Y_2 &= \left[G_1 + \varepsilon(G_2 Y_2 + G_3 Y_2)\right] W(\tau)
\end{aligned}
\tag{II.10}
$$

These are two non-homogeneous stochastic differential equations which are coupled through the parametric terms $Y_1 W(\tau)$ and $Y_2 W(\tau)$. The prime denotes differentiation with respect to the non-dimensional time τ. The coefficients E_i, G_i are defined in Appendix A.

II. 3 Response Moment Equations

The Fokker-Plank equation or the Itô stochastic calculus can be used to generate the differential equations of the response moment as outlined by Ibrahim [4].

Introducing the state variable transformation

$$
\begin{Bmatrix} Y_1 \\ Y_2 \\ Y_1' \\ Y_2' \end{Bmatrix} = \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{Bmatrix}
\tag{II.11}
$$

equations (II.10) can be written in the state Markov form

$$
\begin{aligned}
X_1' &= X_3 \\
X_2' &= X_4 \\
X_3' &= -X_1 - 2\zeta_1 X_3 + (E_1 + \varepsilon E_2 X_1 + \varepsilon E_3 X_2)\, W(\tau) \\
X_4' &= -r^2 X_2 - 2\zeta_2 r X_4 + (G_1 + \varepsilon G_2 X_1 + \varepsilon G_3 X_2)\, W(\tau)
\end{aligned}
\tag{II.12}
$$

The non-stationary Fokker-Planck equation of the evolution of the probability density of the response vector $\underset{\sim}{X}$ is

$$\frac{\partial p}{\partial \tau}(\underset{\sim}{X},\tau) = -\frac{\partial}{\partial X_1}\{X_3 p(\underset{\sim}{X},\tau)\} - \frac{\partial}{\partial X_2}\{X_4 p(\underset{\sim}{X},\tau)\}$$

$$-\frac{\partial}{\partial X_3}\{[-X_1 - 2\zeta_1 X_3]\, p(\underset{\sim}{X},\tau)\}$$

$$-\frac{\partial}{\partial X_4}\{[-r^2 X_2 - 2\zeta_2 r X_4]\, p(\underset{\sim}{X},\tau)\}$$

$$+ D\frac{\partial^2}{\partial X_3^2}\{[E_1^2 + 2\varepsilon E_1E_2X_1 + 2\varepsilon E_1E_3X_2 + \varepsilon^2E_2^2X_1^2$$

$$+ 2\varepsilon^2E_2E_3X_1X_2 + \varepsilon^2E_3^2X_2^2]\; p(\underset{\sim}{X},\tau)\} \tag{II.13}$$

$$+ 2D\frac{\partial^2}{\partial X_3 \partial X_4}\{[E_1G_1 + \varepsilon(E_1G_2 + E_2G_1)X_1 + \varepsilon(E_1G_3 + E_3G_1)X_2$$

$$+ \varepsilon^2E_2G_2X_1^2 + \varepsilon^2(E_2G_3 + E_3G_2)X_1X_2 + \varepsilon^2E_3G_3X_2^2]\, p(\underset{\sim}{X},\tau)\}$$

$$+ D\frac{\partial^2}{\partial X_4^2}\{[G_1^2 + 2\varepsilon G_1G_2X_1 + 2\varepsilon G_1G_3X_2 + \varepsilon^2G_2^2X_1^2$$

$$+ 2\varepsilon^2G_2G_3X_1X_2 + \varepsilon^2G_3^2X_2^2]\, p(\underset{\sim}{X},\tau)\}$$

The following notation for the response joint moment

$$m_{ijk\ell} = \iiiint_{-\infty}^{\infty} X_1^i X_2^j X_3^k X_4^\ell \; p(\underset{\sim}{X},\tau)\,dX_1\,dX_2\,dX_3\,dX_4$$

$$= E[X_1^i X_2^j X_3^k X_4^\ell] \tag{II.14}$$

will be adopted.

Premultiplying both sides of equation (II.13) by $X_1^i X_2^j X_3^k X_4^\ell$ and integrating both sides over the whole space $-\infty < \underset{\sim}{X} < \infty$ gives the following general moment differential equation:

$$
\begin{aligned}
m'_{ijk\ell} = {} & i m_{i-1,j,k+1,\ell} + j m_{i,j-1,k,\ell+1} \\
& - k(m_{i+1,j,k-1,\ell} + 2\zeta_1 m_{i,j,k,\ell}) \\
& - \ell(r^2 m_{i,j+1,k,\ell-1} + 2\zeta_2 r m_{i,j,k,\ell}) \\
& + k(k-1)D\,\big(E_1^2 m_{i,j,k-2,\ell} + 2\varepsilon E_1 E_2 m_{i+1,j,k-2,\ell} \\
& \quad + 2\varepsilon E_1 E_3 m_{i,j+1,k-2,\ell} + \varepsilon^2 E_2^2 m_{i+2,j,k-2,\ell} \\
& \quad + 2\varepsilon^2 E_2 E_3 m_{i+1,j+1,k-2,\ell} + \varepsilon^2 E_3^2 m_{i,j+2,k-2,\ell}\big) \\
& + 2k\ell D\big[E_1 G_1 m_{i,j,k-1,\ell-1} + (E_1 G_2 + E_2 G_1) m_{i+1,j,k-1,\ell-1} \\
& \quad + \varepsilon(E_1 G_3 + E_3 G_1) m_{i,j+1,k-1,\ell-1} + \varepsilon^2 E_2 G_2 m_{i+2,j,k-1,\ell-1} \\
& \quad + \varepsilon^2(E_2 G_3 + E_3 G_2) m_{i+1,j+1,k-1,\ell-1} + \varepsilon^2 E_3 G_3 m_{i,j+2,k-1,\ell-1} \\
& \quad + \varepsilon^2 E_3 G_3 m_{i,j+2,k-1,\ell-1}\big] \\
& + \ell(\ell-1)D\,\big[G_1^2 m_{i,j,k,\ell-2} + 2\varepsilon G_1 G_2 m_{i+1,j,k,\ell-2} \\
& \quad + 2\varepsilon G_1 G_3 m_{i,j+1,k,\ell-2} + \varepsilon^2 G_2^2 m_{i+2,j,k,\ell-2} \\
& \quad + 2\varepsilon^2 G_2 G_3 m_{i+1,j+1,k,\ell-2} + \varepsilon^2 G_3^2 m_{i,j+2,k,\ell-2}\big]
\end{aligned}
\tag{II.15}
$$

An inspection of equation (II.15) shows that the moment equations of any order are consistent, that is, the moment equations of order n are not coupled with moments of order greater than n and they can be solved analytically.

The first and second order response moments are of main concern in random response analysis. The first and second order moment equations are generated from (II.15) and the following stationary solution is obtained:

$$
\begin{aligned}
& m_{1000} = m_{0100} = m_{0010} = m_{0001} = 0 \\
& m_{1010} = m_{0101} = 0 \\
& m_{0002} = r^2 m_{0200} \\
& m_{0110} = -m_{1001} \\
& m_{1001} = \frac{1 - r^2}{2(\zeta_1 + r\zeta_2)} m_{1100}
\end{aligned}
\tag{II.16}
$$

m_{2000}, m_{0200}, m_{1100} are given by the solution of these three equations:

$$
\begin{bmatrix} d_1 & d_2 & d_3 \\ e_1 & e_2 & e_3 \\ f_1 & f_2 & f_3 \end{bmatrix}
\begin{Bmatrix} m_{2000} \\ m_{0200} \\ m_{1100} \end{Bmatrix}
=
\begin{Bmatrix} d_4 \\ e_4 \\ f_4 \end{Bmatrix}
\tag{II.17}
$$

where d_i, e_i, f_i are defined in Appendix B.

The solution of equations (II.17) for m_{2000} and m_{0200} is shown in Fig. II.3 as a function of the frequency ratio ω_{22}/ω_{11} for various values of the mass parameter μ. The response mean square (in terms of the generalized coordinates) is obtained through the inverse modal transformation and is plotted in Fig. II.4. Figures II.3 and II.4 reveal that the mean square

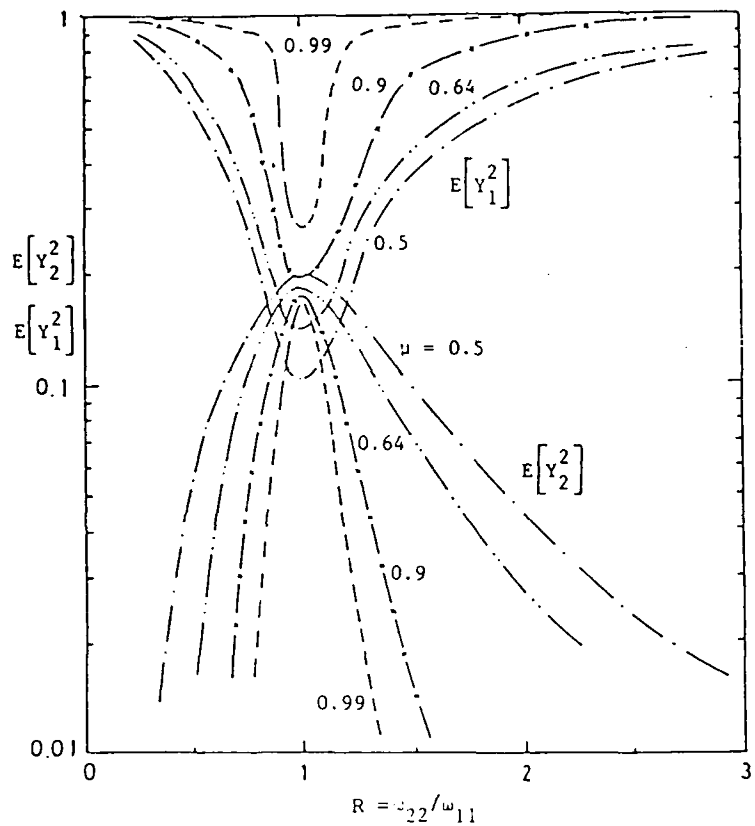


Fig. II.3 Mean squares of the normal coordinates response against the frequency ratio R for various values of mass parameter μ.

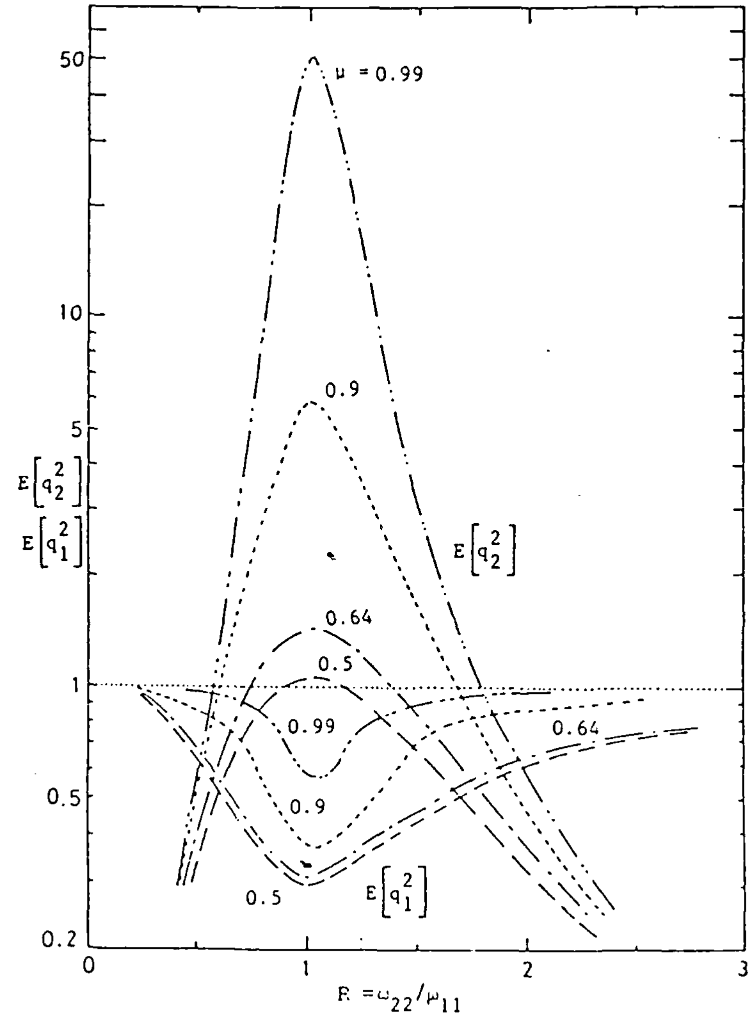


Fig. II.4 Mean square response of the generalized coordinates q_1 and q_2 against the system frequency ratio R for various values of mass parameter μ.

response exhibits a strong interaction in the neighborhood of the frequency ratio $\omega_{22}/\omega_{11} = 1$. The degree of interaction depends on the mass parameter μ which measures the degree of linear dynamic coupling. The interaction of the two modes exhibits the linear vibration absorber characteristics known in the deterministic vibration. It is also seen that as the mass parameter μ diminishes, the absorbing effect increases. For modest values of $\mu < 1$, the first mode Y_1 is suppressed and the second mode Y_2 reaches its peak value. Furthermore, the response of the main horizontal beam is that of a single-degree-of-freedom system outside the frequency ratio $\omega_{22}/\omega_{11} = 1 \pm O(\varepsilon)$, and the range of this frequency ratio is influenced by the mass ratio parameter μ as shown in Fig. II.4.

In order to examine the influence of parametric random excitation on the system response, the stationary solution was obtained by setting ℓ_{ij}, $i = 1,2$ to zero from the moment equations. Figures II.5 and II.6 show the mean square response in terms of the normal coordinates and generalized coordinates, respectively. It is seen that the parametric excitation has very negligible effect on the overall response level. However, it is the system stochastic stability which is governed by the parametric excitation [4].

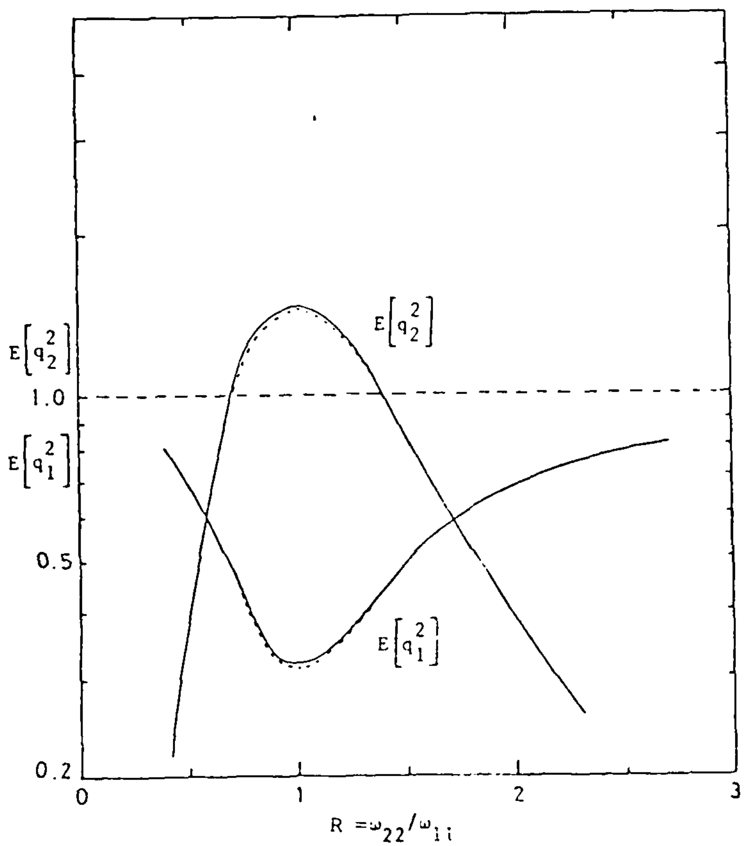


Fig. II.5 Effect of the random parametric excitation on $E\left[q_1^2\right]$ and $E\left[q_2^2\right]$ for mass parameter $\mu = 0.64$.

__________ : external and parametric excitation

\- - - - - - - : external excitation

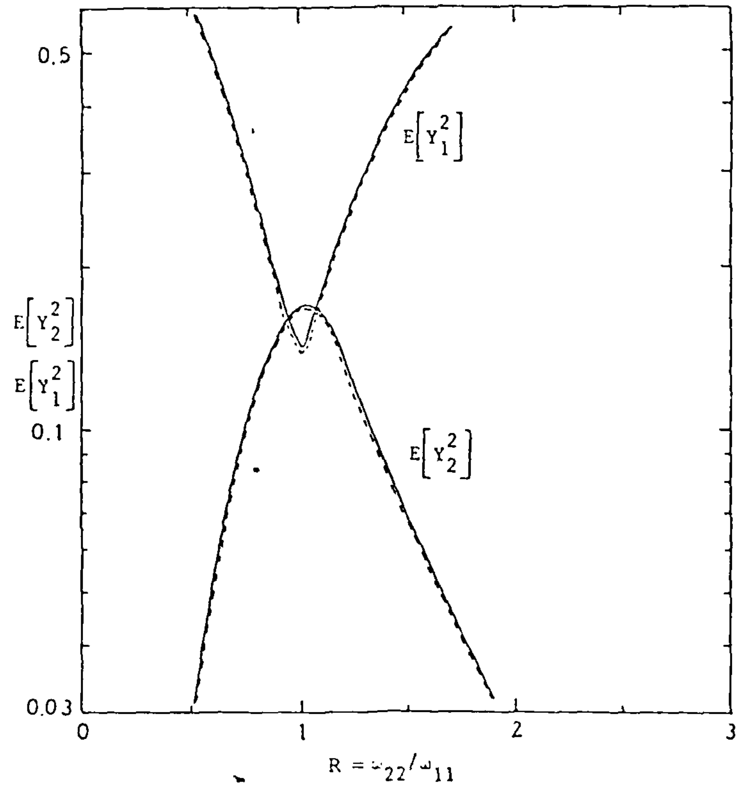


Fig.II.6 Effect of random parametric excitation of the normal response mean squares under random external excitation for $\mu = 0.64$.

__________ : external and parametric excitation

\- - - - - - - : external excitation

CHAPTER III

NON-LINEAR ANALYSIS

III.1 Introduction

The linear analysis of the coupled aeroelastic structure examined in Chapter 2 may be sufficient to describe the dynamic behavior of the system as long as its motion is very close to the static equilibrium position. However, the structure may experience new types of modal interactions when the non-linear effects become significant. Of particular importance is the non-linear coupling of the system normal modes. It is well known in the deterministic theory of non-linear oscillations [5] that this type of non-linearity may give rise to the dynamic instability known as "internal resonance." Methods for studying non-linear effects in random vibration problems involve a number of difficulties in determining the response dynamic characteristics [4]. These difficulties include the solution of the system Fokker-Planck equation and the problem of infinite coupled moment equations of the response coordinates.

Over the past few decades a number of closure schemes have been developed [6]. In the present analysis two closure schemes will be proposed to close the moment dynamic equations of the response coordinates. The first scheme is based on the assumption that the response process is Gaussian distributed under Gaussian excitation. However, the application of this method to non-linear systems is not mathematically justified since the response is non-Gaussian. Therefore, it is assumed that the excitation is of small intensity and the response will not depart significantly from normality. The response process can then be described adequately in terms of

the first and second order cumulants (semi-invariants), and all higher order cumulants vanish. Cumulants are statistical functions which can be related to the moments. For example, first and second order cumulants are equivalent to the mean and variance of the response, respectively.

The second method is more accurate and includes the effects of the non-normality of the response. Unlike Gaussian-closure methods, cumulants of order higher than two will not vanish and will give a measure of the deviation of the response from normality. To the first order approximation third and fourth order cumulants will be significant to account for the response non-normality while the fifth order cumulant will contribute less and can be neglected. Consequently, one can express the fifth order joint moments in terms of lower order moments. Hitherto, the present approach has not been applied to multi-degree-of-freedom non-linear systems.

It is expected that new features of the response characteristics may be obtained due to the system's inherent non-linearity. The influence of the internal resonance condition will be examined for various values of the system parameters.

III.2 Theoretical Analysis

The non-linear equations of motion of the system (2.5), shown in Fig. I.1, are

$$\begin{bmatrix} m_1 + m_2\left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2\right) & 1.5m_2\frac{\ell_2}{\ell_1} \\ 1.5m_2\frac{\ell_2}{\ell_1} & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

$$= -\ddot{\xi}(t)\begin{Bmatrix} m_1 + m_2 \\ 0 \end{Bmatrix} - \ddot{\xi}(t)\begin{bmatrix} 2.25m_2\frac{\ell_2}{\ell_1^2} & 1.5m_2\frac{1}{\ell_1} \\ 1.5m_2\frac{1}{\ell_1} & 1.2m_2\frac{1}{\ell_2} \end{bmatrix}\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} - m_2\begin{Bmatrix} \psi_1 \\ \psi_2 \end{Bmatrix} \qquad \text{(III.1)}$$

where

$$\psi_1 = 0.9 \frac{\ell_2}{\ell_1^2} q_1\ddot{q}_1 + 0.45 \frac{\ell_2}{\ell_1^2} \dot{q}_1^2 + \frac{1.2}{\ell_2}\left(q_2\ddot{q}_2 + \dot{q}_2^2\right) + \frac{0.3}{\ell_1} q_1\ddot{q}_2$$

$$+ \frac{3}{\ell_1}\left(\ddot{q}_1 q_2 + \dot{q}_1\dot{q}_2\right)$$

$$\psi_2 = \frac{0.3}{\ell_1} q_1\ddot{q}_1 - \frac{1.2}{\ell_1}\dot{q}_1^2 + \frac{1.2}{\ell_2} q_2\ddot{q}_1$$

$$k_i = \frac{3E_i I_i}{\ell_i^3} \qquad i = 1,2$$

Equations (III.1) can be written in terms of normal coordinates y_i via the transformation

$$\{q\} = [R]\,\{y\} \tag{III.2}$$

where modal matrix [R] was defined in section II.2.

Premultiplying equation (III.1) by $[R]^{-1}\,[m]^{-1}$, where [m] is the mass matrix and using transformation (III.2), the equations of motion take the form

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} Y_1'' \\ Y_2'' \end{Bmatrix} + \begin{bmatrix} 2\zeta_1 & 0 \\ 0 & 2\zeta_2 r \end{bmatrix}\begin{Bmatrix} Y_1' \\ Y_2' \end{Bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix}\begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix}$$

$$= \xi''(\tau)\begin{Bmatrix} a_1 \\ b_1 \end{Bmatrix} + \varepsilon\xi''(\tau)\begin{bmatrix} a_2 & a_3 \\ b_2 & b_3 \end{bmatrix}\begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix} + \varepsilon\begin{Bmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \end{Bmatrix} \tag{III.3}$$

where prime denotes differentiation with respect to the time parameter $\tau = \omega_1 t$ and $r = \omega_2/\omega_1$. A linear viscous damping has been introduced to account for energy dissipation of the system. The following non-dimensional parameters have been used:

$$\varepsilon = \frac{q_1^o}{\ell_1} \tag{III.4}$$

$$\{Y_1, Y_2\} = \{y_1, y_2\}/q_1^o$$

q_1^o is the response mean square of the system, which was defined in section II.2, and the non-linear functions $\bar{\psi}_1$ and $\bar{\psi}_2$ are

$$\begin{aligned}\bar{\psi}_1 &= a_4 Y_1 \ddot{Y}_1 + a_5 Y_1 \ddot{Y}_2 + a_6 Y_2 \ddot{Y}_1 + a_7 Y_2 \ddot{Y}_2 + a_8 Y_1'^2 + a_9 Y_1' Y_2' + a_{10} Y_2'^2 \\ \bar{\psi}_2 &= b_4 Y_1 \ddot{Y}_1 + b_5 Y_1 \ddot{Y}_2 + b_6 Y_2 \ddot{Y}_1 + b_7 Y_2 \ddot{Y}_2 + b_8 Y_1'^2 + b_9 Y_1' Y_2' + b_{10} Y_2'^2\end{aligned} \tag{III.5}$$

where the coefficients a_i and b_i are given in Appendix C. Two types of non-linearity are embodied in $\bar{\psi}_1$, $\bar{\psi}_2$. The first forms the non-linear terms of the same mode such as $Y_1 \ddot{Y}_1$ or $Y_1'^2$ in the first equations of mode 1, and the second constitutes autoparametric terms such as $Y_2 \ddot{Y}_1$ (non-linear coupling). Autoparametric terms give rise to $r = \frac{\omega_2}{\omega_1} = 0.5$, (here $\omega_2 < \omega_1$, where the order of modes is reversed).

The acceleration $\xi''(\tau)$ is assumed to be a Gaussian wide band random process with zero mean and a smooth spectral density 2D up to some frequency which is higher than any characteristic frequency of the system.

In order to represent the response coordinates as a Markov process the acceleration $\ddot{Y}_i$ associated with the non-linear terms must be removed by successive elimination. Having eliminated $\ddot{Y}_i$ from the non-linear terms in equations (III.3) the following coordinate transformation is introduced:

$$\begin{Bmatrix} Y_1 \\ Y_2 \\ Y_1' \\ Y_2' \end{Bmatrix} = \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{Bmatrix} \tag{III.6}$$

Equations (III.3) can be written in the Markov vector form:

$$\begin{aligned}
X_1' &= X_3 \\
X_2' &= X_4 \\
X_3' &= - X_1 - 2\zeta_1 X_3 - a_4 X_1^2 + \left(a_6 + r^2 a_5\right) X_1 X_2 - r^2 a_7 X_2^2 \\
&\quad - 2\zeta_1 a_4 X_1 X_3 - 2\zeta_2 r a_5 X_1 X_4 - 2\zeta_1 a_6 X_2 X_3 - 2\zeta_2 r a_7 X_2 X_4 + a_8 X_3^2 \\
&\quad + a_9 X_3 X_4 + a_{10} X_4^2 - \left(A_1 + A_2 X_1 + A_3 X_2 + A_4 X_1^2 + A_5 X_1 X_2 + A_6 X_2^2\right) W(\tau) \\
X_4' &= - r^2 X_2 - 2\zeta_2 r X_4 - b_4 X_1^2 - \left(b_6 + r^2 b_5\right) X_1 X_2 - r^2 b_7 X_2^2 - 2\zeta_1 b_4 X_1 X_3 \\
&\quad - 2\zeta_2 r b_5 X_1 X_4 - 2\zeta_1 b_6 X_2 X_3 - 2\zeta_2 r b_7 X_2 X_4 + b_8 X_3^2 + b_9 X_3 X_4 + b_{10} X_4^2 \\
&\quad - \left(B_1 + B_2 X_1 + B_3 X_2 + B_4 X_1^2 + B_5 X_1 X_2 + B_6 X_2^2\right) W(\tau)
\end{aligned} \tag{III.7}$$

where the coefficients A_i and B_i are given in Appendix D.

In the equations (III.3) the random acceleration $\xi''(\tau)$ has been replaced by the white noise process $W(\tau)$, where the Wong-Zakai [7] correction term is zero. The autocorrelation function of $W(\tau)$ is defined by the relationship

$$R[\Delta\tau] = E[W(\tau)W(\tau + \Delta\tau)] = 2D\,\delta(\Delta\tau) \tag{III.8}$$

where 2D is the spectral density, and $\delta(\ )$ is the Dirac delta function.

A general differential equation for all possible moments can be generated by using the Itô stochastic calculus or the Fokker-Planck equation [4]. Following the same procedure described in Section II.3 the general differential equation of the response joint moment is

$$
\begin{aligned}
m'_{i,j,k,\ell} = {} & i m_{i-1,j,k+1,\ell} + j m_{i,j-1,k,\ell+1} \\
& + k\Big[- m_{i+1,j,k-1,\ell} - 2\zeta_1 m_{i,j,k,\ell} - a_4 m_{i+2,j,k-1,\ell} \\
& - \left(r^2 a_5 + a_6\right) m_{i+1,j+1,k-1,\ell} - r^2 a_7 m_{i,j+2,k-1,\ell} \\
& - 2\zeta_1 a_4 m_{i+1,j,k,\ell} - 2\zeta_2 r a_5 m_{i+1,j,k-1,\ell+1} - 2\zeta_1 a_6 m_{i,j+1,k,\ell} \\
& - 2\zeta_2 r a_7 m_{i,j+1,k-1,\ell+1} + a_8 m_{i,j,k+1,\ell} + a_9 m_{i,j,k,\ell+1} \\
& + a_{10} m_{i,j,k-1,\ell+2}\Big] + \ell\Big[- r^2 m_{i,j+1,k,\ell-1} - 2\zeta_2 r m_{i,j,k,\ell} \\
& - b_4 m_{i+2,j,k,\ell-1} - \left(r^2 b_5 + b_6\right) m_{i+1,j+1,k,\ell-1} \\
& - r^2 b_7 m_{i,j+2,k,\ell-1} - 2\zeta_1 b_4 m_{i+1,j,k+1,\ell-1} \\
& - 2\zeta_2 r b_5 m_{i+1,j,k,\ell} - 2\zeta_1 b_6 m_{i,j+1,k+1,\ell-1} - 2\zeta_2 r b_7 m_{i,j+1,k,\ell} \\
& + b_8 m_{i,j,k+2,\ell-1} + b_9 m_{i,j,k+1,\ell} + b_{10} m_{i,j,k,\ell+1}\Big] \\
& + k\,(k-1)\Big[D A_1^2 m_{i,j,k-2,\ell} + 2 D A_1 A_2 m_{i+1,j,k-2,\ell} \\
& + 2 D A_1 A_3 m_{i,j+1,k-2,\ell} + D\left(2A_1 A_4 + A_2^2\right) m_{i+2,j,k-2,\ell} \\
& + 2D(A_1 A_5 + A_2 A_3) m_{i+1,j+1,k-2,\ell} + D\left(A_3^2 + 2A_1 A_6\right) m_{i,j+2,k-2,\ell}\Big] \\
& + k\ell\Big[2 D A_1 B_1 m_{i,j,k-1,\ell-1} + 2D\left(A_1 B_2 + A_2 B_1\right) m_{i+1,j,k-1,\ell-1} \\
& + 2D\left(A_1 B_3 + A_3 B_1\right) m_{i,j+1,k-1,\ell-1}
\end{aligned}
\qquad \text{(III.9)}
$$

$$+ 2D(A_1B_4 + A_4B_1 + A_2B_2)m_{i+2,j,k-1,\ell-1}$$

$$+ 2D(A_1B_5 + A_5B_1 + A_2B_3 + A_3B_2)m_{i+1,j+1,k-1,\ell-1}$$

$$+ 2D(A_3B_3 + A_1B_6 + A_6B_1)m_{i,j+2,k-1,\ell-1}]$$

$$+ \ell(\ell-1)[DB_1^2 m_{i,j,k,\ell-2} + 2DB_1B_2 m_{i+1,j,k,\ell-2}$$

$$+ 2DB_1B_3 m_{i,j+1,k,\ell-2} + D(2B_1B_4 + B_2^2)m_{i+2,j,k,\ell-2}$$

$$+ 2D(B_1B_5 + B_2B_3)m_{i+1,j+1,k,\ell-2} + D(B_3^2 + 2B_1B_6)m_{i,j+2,k,\ell-2}]$$

(III.9)
cont'd.

It is seen that a moment equation of order $n = i+j+k+\ell$ contains moments of order n and $n+1$, thus forming an "infinite hierarchy set." In order to close the moment equations the following two closure schemes will be used.

III.3 Gaussian Closure Solution

From equation (III.9) it is possible to generate four equations for the first order moments and ten equations for the second order moments. These equations are, however, coupled through third order moment terms.

Given the assumption that the system non-linearities are too small to the extent that the response can be regarded as "nearly" Gaussian, then the fourteen equations can be closed by using the Gaussian cumulant-neglect scheme [4]. Under this assumption the cubic semi-invariants vanish and the third order moment terms can be expressed in terms of lower order moments, i.e.,

$$\lambda_3[X_iX_jX_k] = E[X_iX_jX_k] - \sum^{3} E[X_i]E[X_jX_k] + 2E[X_i]E[X_j]E[X_k] = 0 \qquad \text{(III.10)}$$

where the number over the summation sign refers to the number of terms generated by the indicated expression without allowing permutation of indices.

The resulting closed 14 coupled non-linear differential equations are integrated numerically by using IMSL DVERK routine (Runge-Kutta-Verner fifth and sixth order numerical integration method).

Figure III.1 shows the time history of the system mean square response in normal coordinates for internal resonance ratio r = 0.5, damping ratios $\zeta_1 = \zeta_2 = 0.02$, mass ratio $m_2/m_1 = 0.2$, beam length ratio $\ell_2/\ell_1 = 0.6$ and non-linear coupling parameter $\varepsilon = 0.02$. After a sufficient period of time $\tau = 1000$ the response fluctuates between two limits, indicating that the system does not achieve a stationary response.

The effect of damping ratios ζ_1 and ζ_2 on the response mean squares in normal and generalized coordinates is shown in Fig. III.2. It is seen that as the damping ratios decrease, the region of autoparametric interaction becomes wider and the peak of the mean square response of the two modes increases. Also, the quasi-stationarity of the Gaussian closure solutions are manifested over a wider range of $r = \omega_2/\omega_1$.

Figure III.3 shows the effect of the non-linear coupling parameter ε. For very small values of ε, the effect of non-linearities in the equations of motion (III.1) is greatly reduced. The mean square responses do not exhibit any non-linear modal interaction and almost follow the linear solutions. However, it can be seen that the system responds very differently for even minor increases in ε. As ε increases, the interaction region becomes wider and the autoparametric interaction takes place in a form of energy exchange between the two modes.

The effect of mass ratio is plotted in Fig. III.4. As the mass ratio is increased, the region of autoparametric interaction between the displacement mean square responses in normal coordinates shrinks to a narrow range of internal tuning ratio.

Figure III.5 illustrates the effect of length ratio between the two beams on the mean squares of the system response. As the length ratio is increased, the region of autoparametric interaction shrinks to a narrow range of internal tuning ratio.

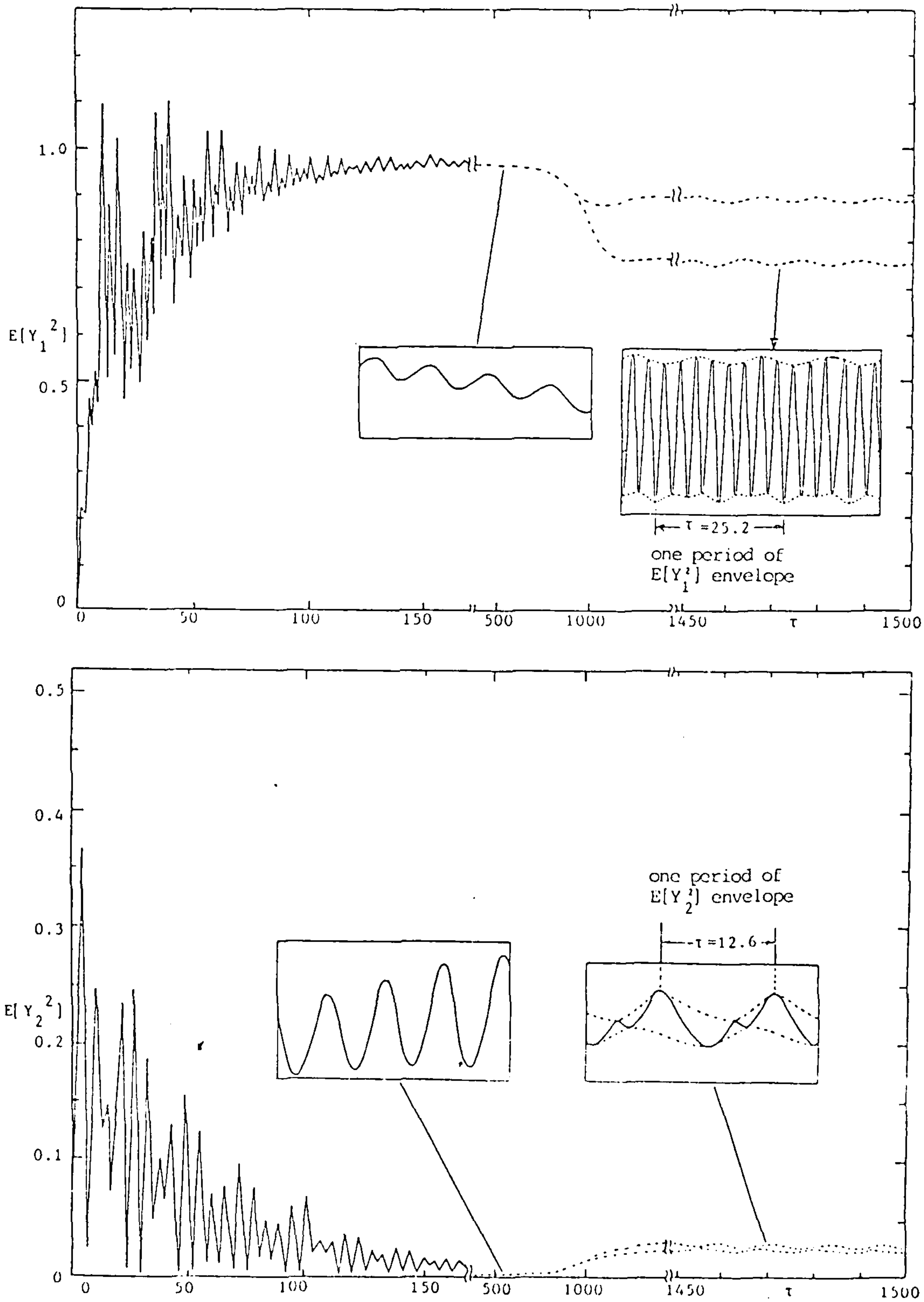
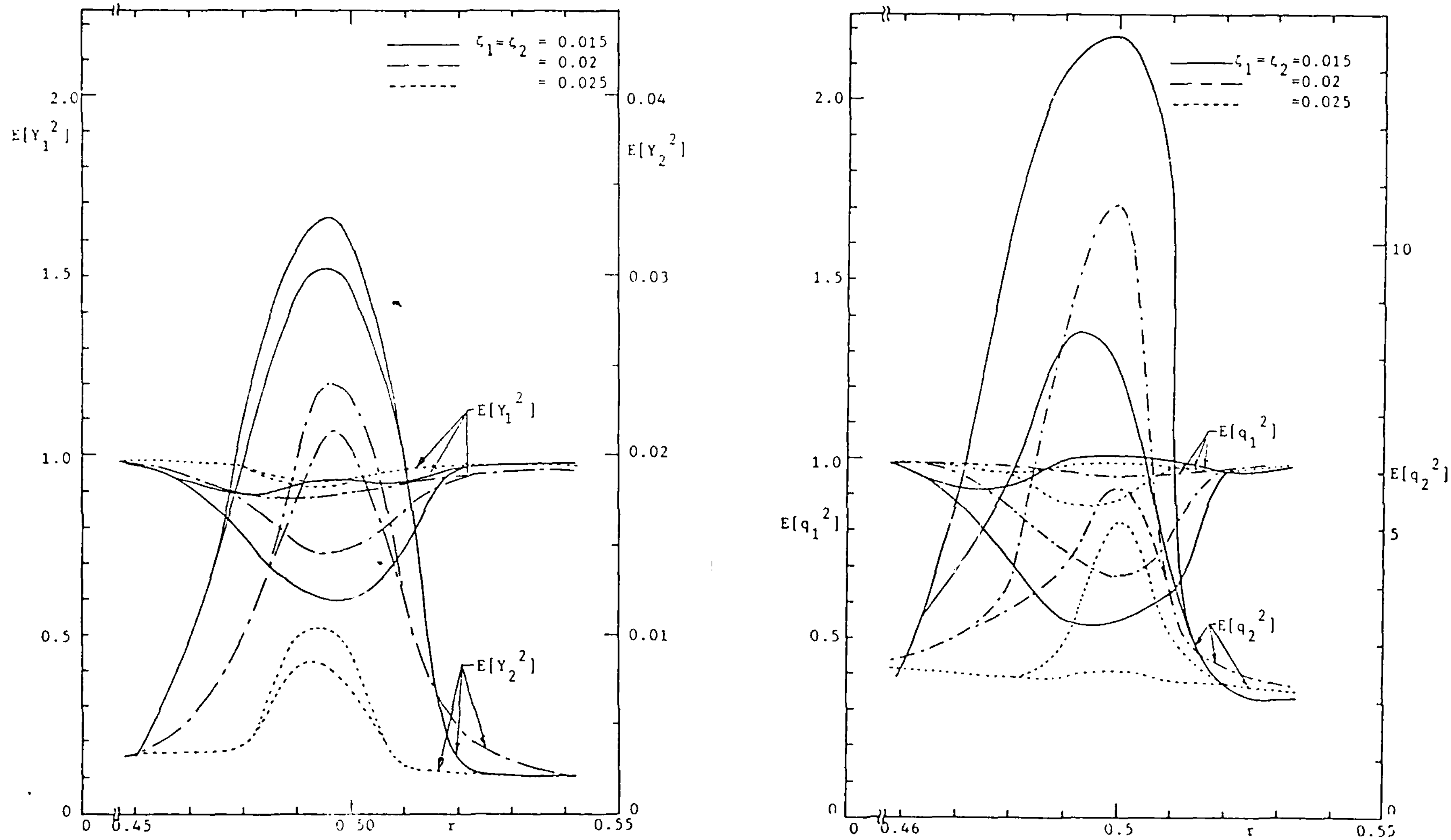


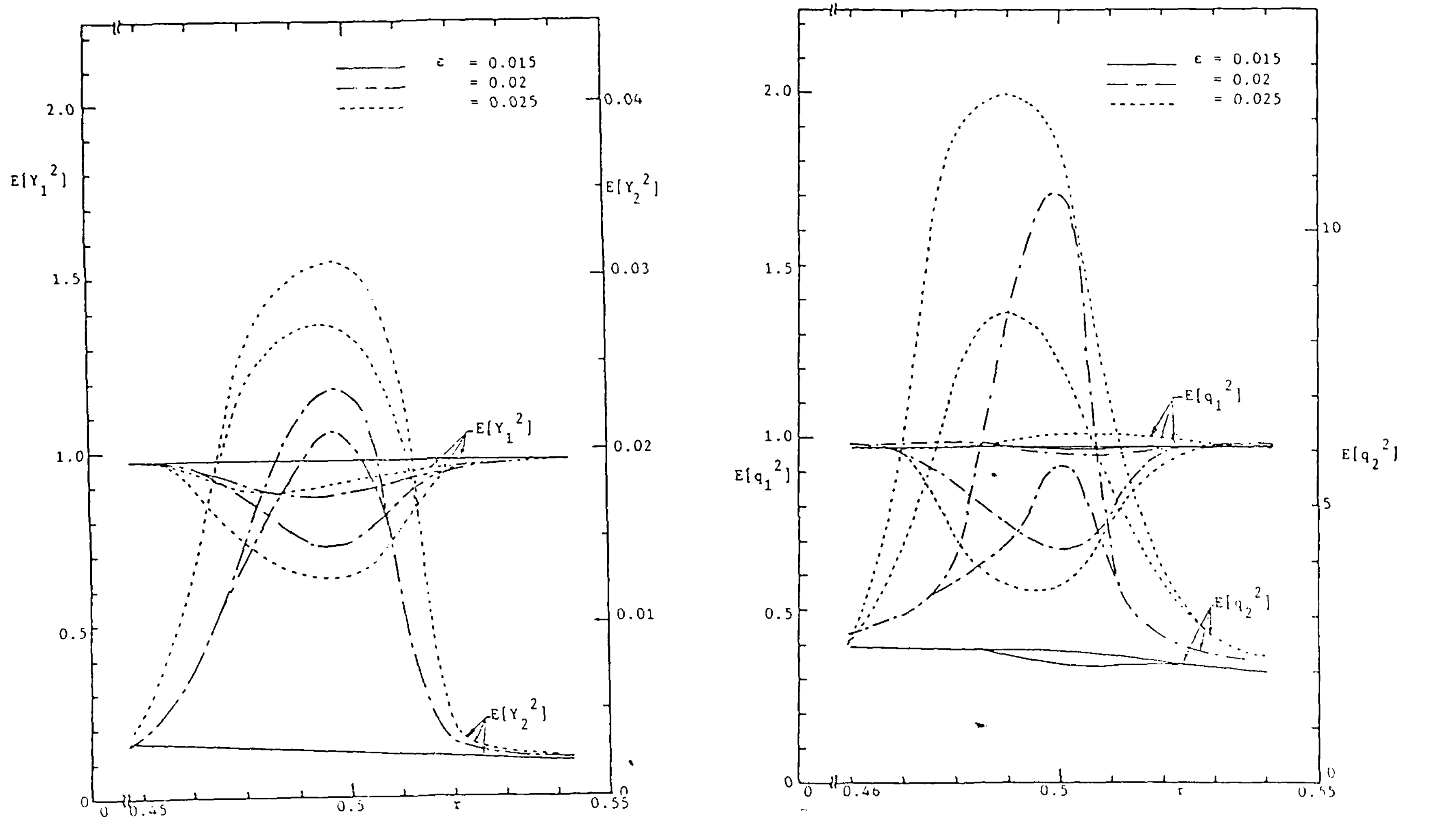
Fig. III.1 Time history of Gaussian response with initial conditions $E[Y_2^2] = 0.1$, $E[Y_1 Y_2 Y_1' Y_2'] = 0.00001$ and the rest = 0.0.



(a) Normal mode mean square response.

(b) Generalized mean square response.

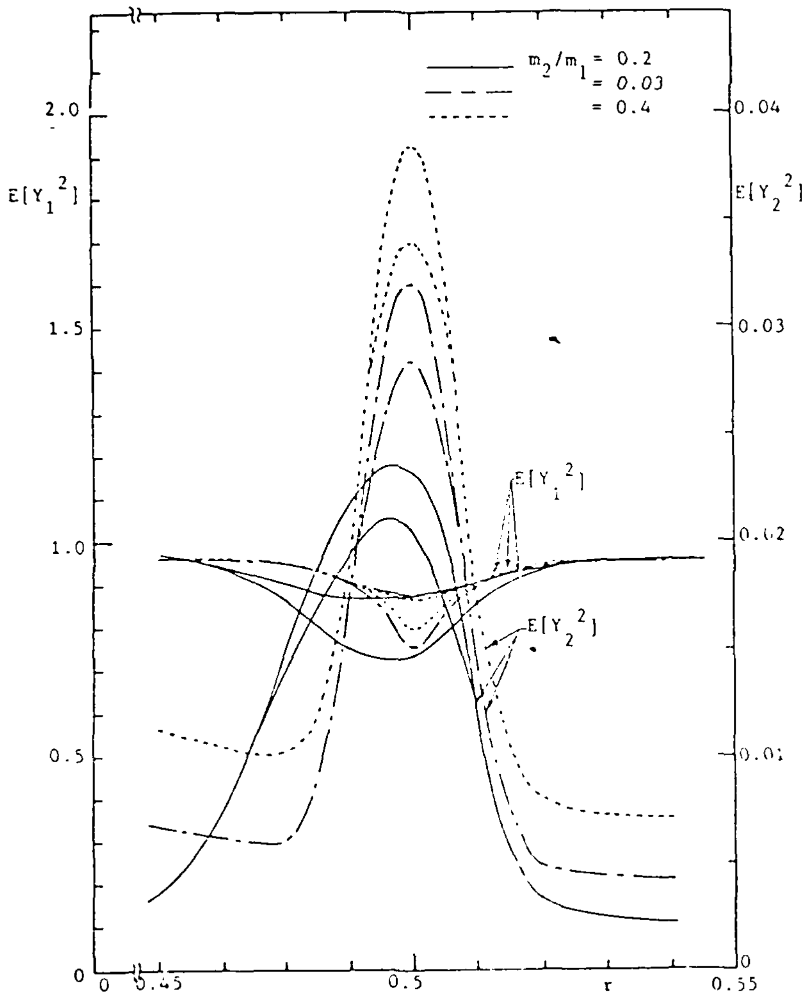
Fig. III.2 Gaussian closure solution for various values of damping ratios.



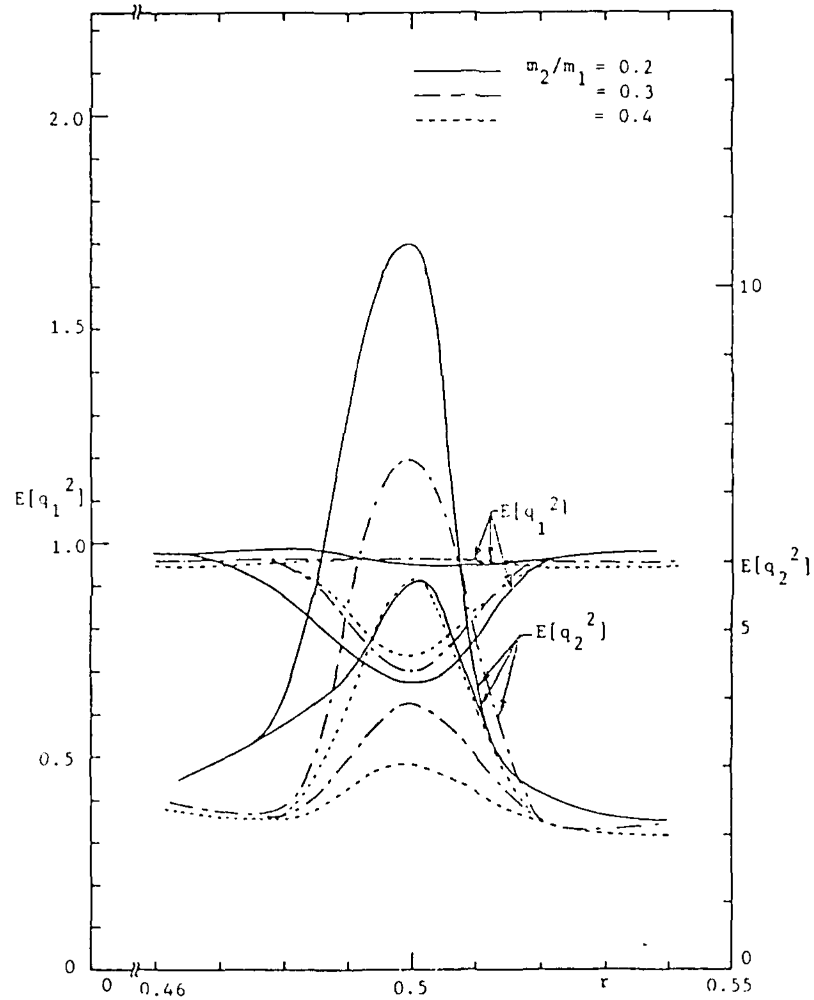
(a) Normal mode mean square response.

(b) Generalized mean square response.

Fig. III.3 Gaussian closure solution for various values of non-linear coupling parameter ε.

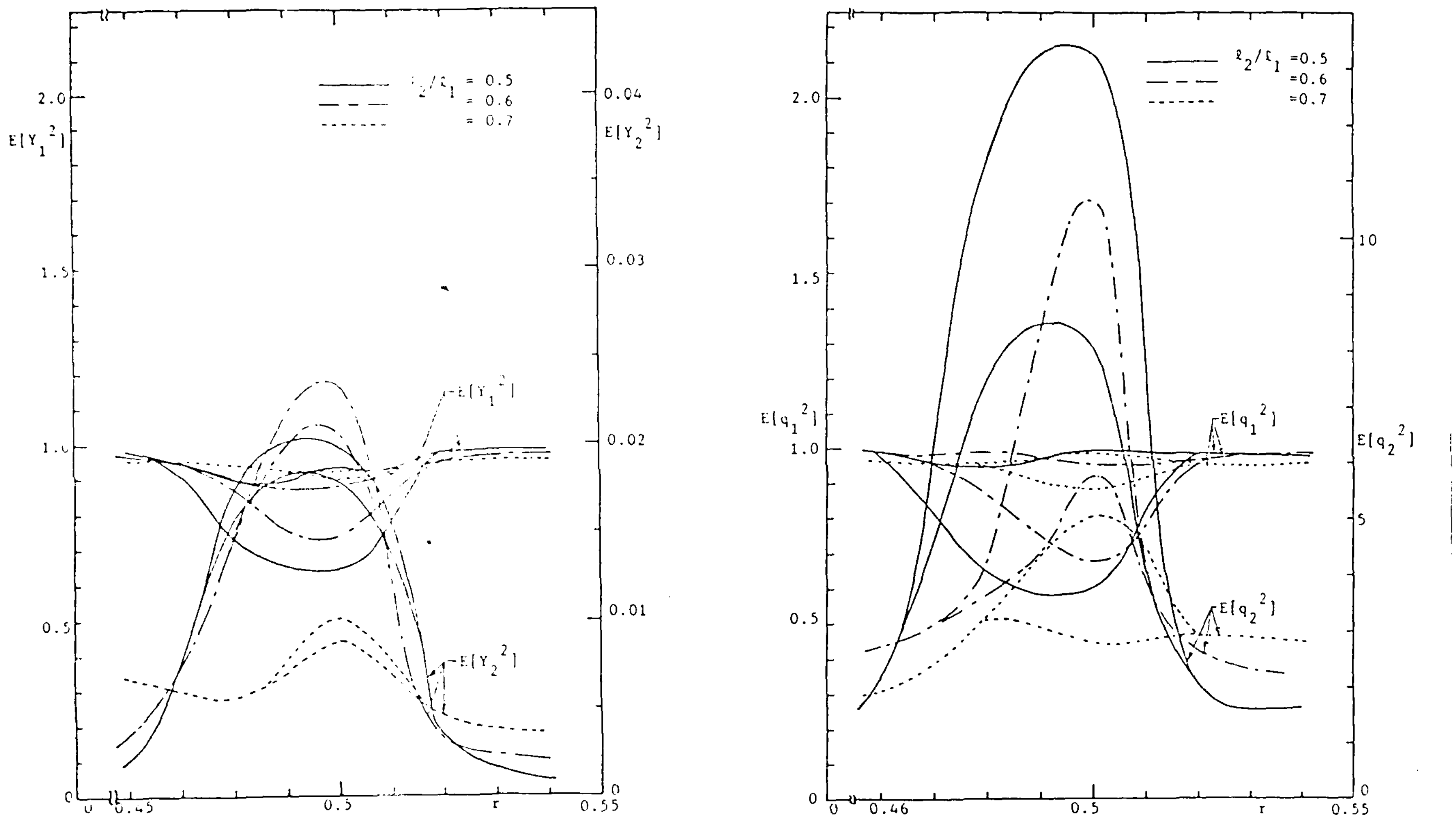


(a) Normal mode mean square response.



(b) Generalized mean square response.

Fig. III.4 Gaussian closure solution for various values of mass ratio.



(a) Normal mode mean square response.

(b) Generalized mean square response.

Fig. III.5 Gaussian closure solution for various values of length ratio.

III.4 Non-Gaussian Closure Solution

In the theory of random processes [8] it is known that any linear operator on a Gaussian process results in another Gaussian process. However, if the system is non-linear or involves random coefficients, then the response will not be Gaussian and the corresponding third and higher order semi-invariants will not vanish. These higher order semi-invariants (cumulants) give a measure to the non-normality of the response distribution. If the response process is assumed to be slightly deviated from Gaussian, the contribution of the higher order cumulants diminishes as the order increases. Under this assumption, a better approximation can be obtained by letting the fifth and higher order cumulants be zero, i.e.,

$$\begin{aligned}\lambda_5[X_iX_jX_kX_\ell X_m] &= E[X_iX_jX_kX_\ell X_m] - \sum^{5} E[X_i]E[X_jX_kX_\ell X_m] \\ &+ 2\sum^{10} E[X_i]E[X_j]E[X_kX_\ell X_m] - 6\sum^{10} E[X_i]E[X_j]E[X_k]E[X_\ell X_m] \\ &+ 2\sum^{15} E[X_i]E[X_jX_k]E[X_\ell X_m] - \sum^{10} E[X_iX_j]E[X_kX_\ell X_m] \\ &+ 24E[X_i]E[X_j]E[X_k]E[X_\ell]E[X_m] = 0\end{aligned} \tag{III.11}$$

Moment differential equations of order up to four will be generated from equation (III.9). Sixty-nine of these equations are coupled through the fifth order moment terms. These 69 equations are closed by using the relation expressed in (III.11).

These equations are integrated numerically by using IMSL DVERK routine (double precision). The time history response of the mean square displacements in normal coordinates are shown in Fig. II.6 for internal resonance

ratio r = 0.5, damping ratios $\zeta_1 = \zeta_2 = 0.02$, mass ratio $m_2/m_1 = 0.2$ and length ratio $\ell_2/\ell_1 = 0.6$. During the transient period the mean square of the first normal mode displacement fluctuates and grows until it reaches a peak value at $\tau = 60$ and then drops to a lower level at $\tau = 110$. During this transient period, the mean square of the second normal mode displacement fluctuates and drops until it reaches its minimum value at $\tau = 60$, then grows not significantly to a peak value at $\tau = 110$. This kind of interaction shows an energy exchange between the two modes during a transient period, after which each mode achieves a complete stationary response.

The stationarity of the solution is confirmed by solving numerically the non-linear algebraic equations resulting from the set of original differential equations. The numerical solution is achieved by using the IMSL routine ZSCNT (Secant method for simultaneous non-linear equations). The algebraic numerical solution is identical to the stationary solution obtained by numerical integration. However, for Gaussian closure, the algebraic solution does not converge for all possible initial guessing values.

Figure III.7 shows the effect of damping ratios on the autoparametric interaction region. It is seen that as the damping ratios decrease the region of autoparametric integration between the two modes broadens and the difference between the peak mean square responses of the two modes increases, but less than in the Gaussian closure solution. Unlike the Gaussian closure solution, the stationarity of the non-Gaussian closure solution is manifested over a wider range of $r = \omega_2/\omega_1$.

The effect of the non-linear coupling parameter ε is shown in Fig. III.8. The small value of ε, which exhibits no non-linear effect in the Gaussian closure solution, shows modal interaction in the non-Gaussian closure solution. Also, as the non-linear coupling parameter ε increases, the region of modal interaction widens and the difference between the two peak values increases.

The effect of the mass ratio on the mean square responses of the system is shown in Fig. III.9. As the mass ratio increases, the mean square of the second mode displacement increases, accompanied by a suppression in the mean square of the first mode. Graphs are shown for both normal and generalized coordinates.

Figure III.10 demonstrates the effect of length ratio on the mean squares of the system response. As the length ratio increases, the mean square of the second mode displacement decreases. This decrease is accompanied by a noticeable increase in the mean square of the first mode displacement.

The effect of initial conditions on the system mean square response is then examined to see if the system possesses more than one limit cycle. When the initial condition of $E[Y_1 Y_2 Y_1' Y_2']$ (= m_{1111}) is set equal to 0.00001 and all other moments have zero initial conditions, the Gaussian closure solution gives the same quasi-stationary response as those solutions obtained with a different set of initial conditions (compare Figs. III.1 and III.11). In the case of the non-Gaussian closure solutions, it is also found that the initial conditions have no effect on the final steady state response, as shown in Figs. III.6 and III.12.

The C.P.U. time for I.B.M. 3033 system required to generate the Gaussian solution up to $\tau = 1500$ is 24 seconds while for the non-Gaussian closure solution, it is 487 seconds for the same response period.

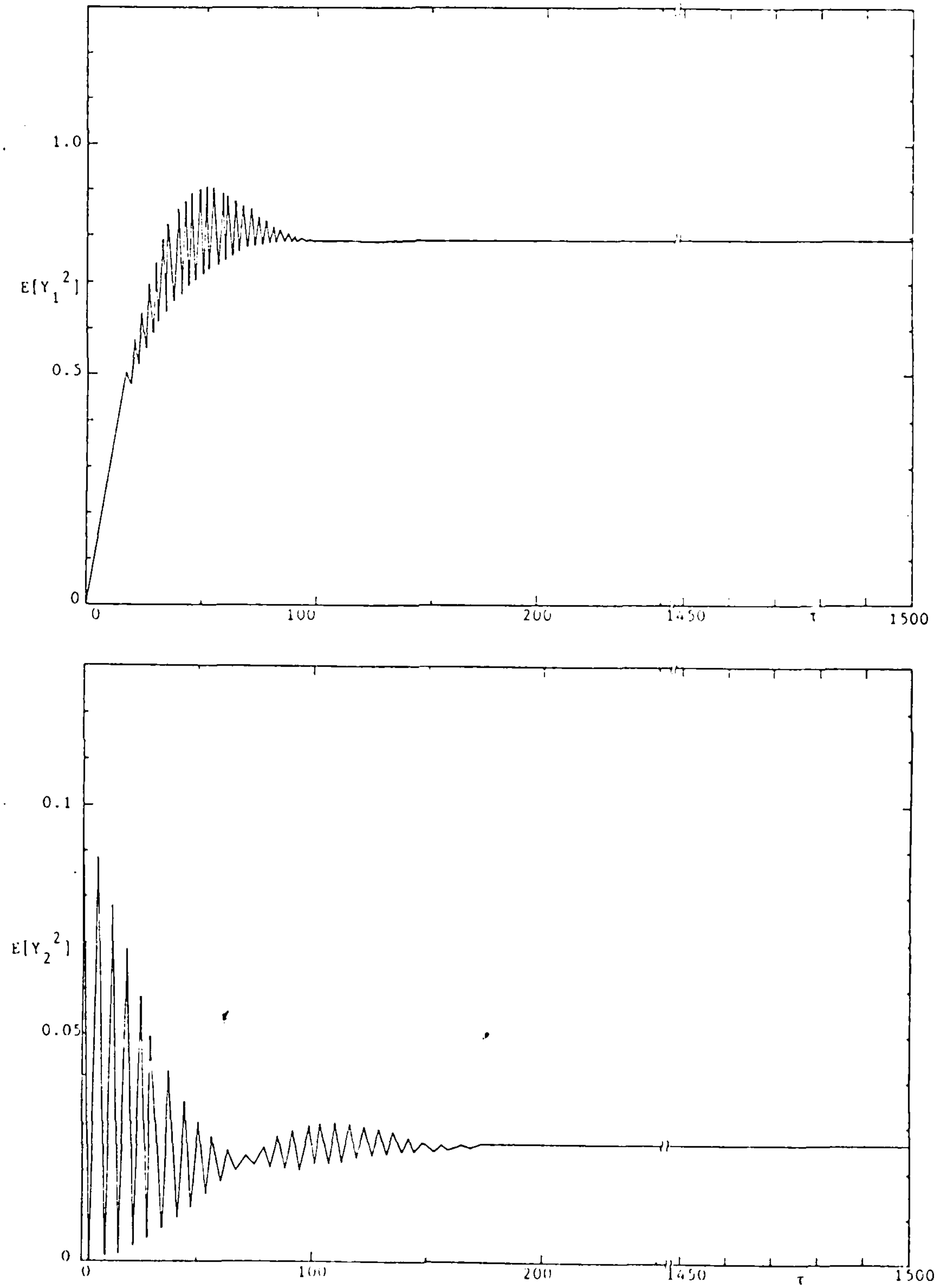
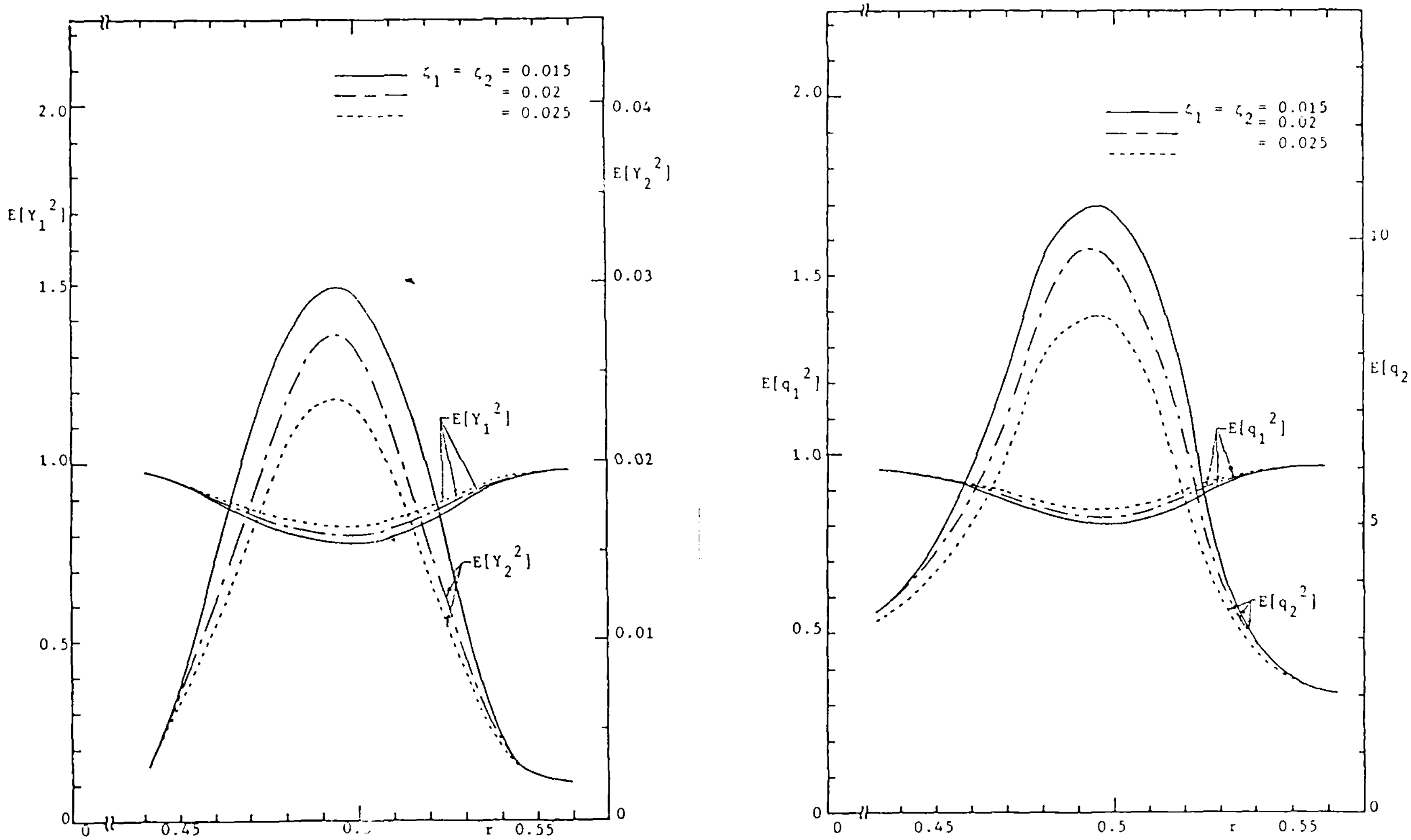


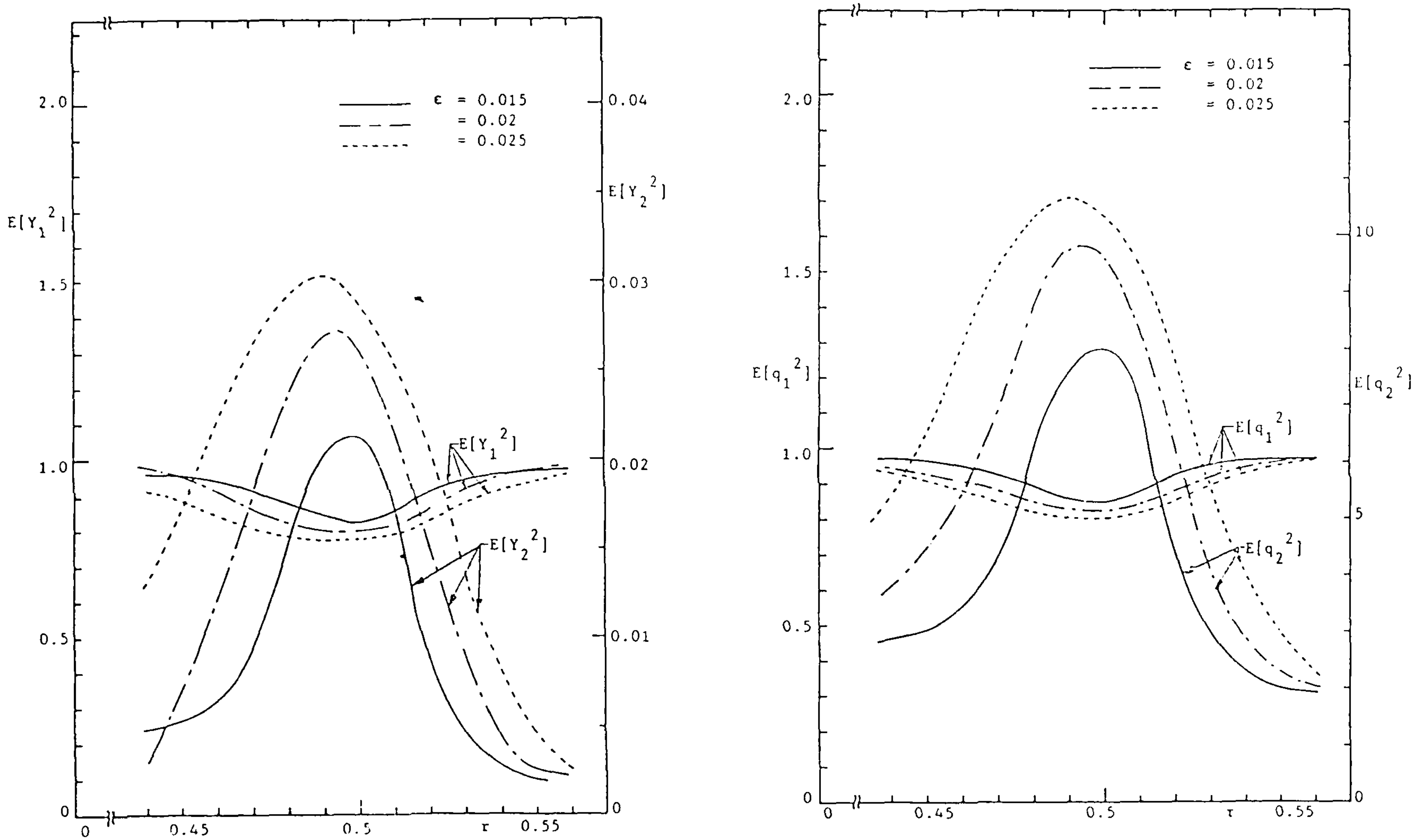
Fig. III.6 Time history of non-Gaussian response with initial conditions $E[Y_2^2] = 0.1$, $E[Y_1 Y_2 Y_1' Y_2'] = 0.00001$ and the rest = 0.0.



(a) Normal mode mean square response.

(b) Generalized mean square response.

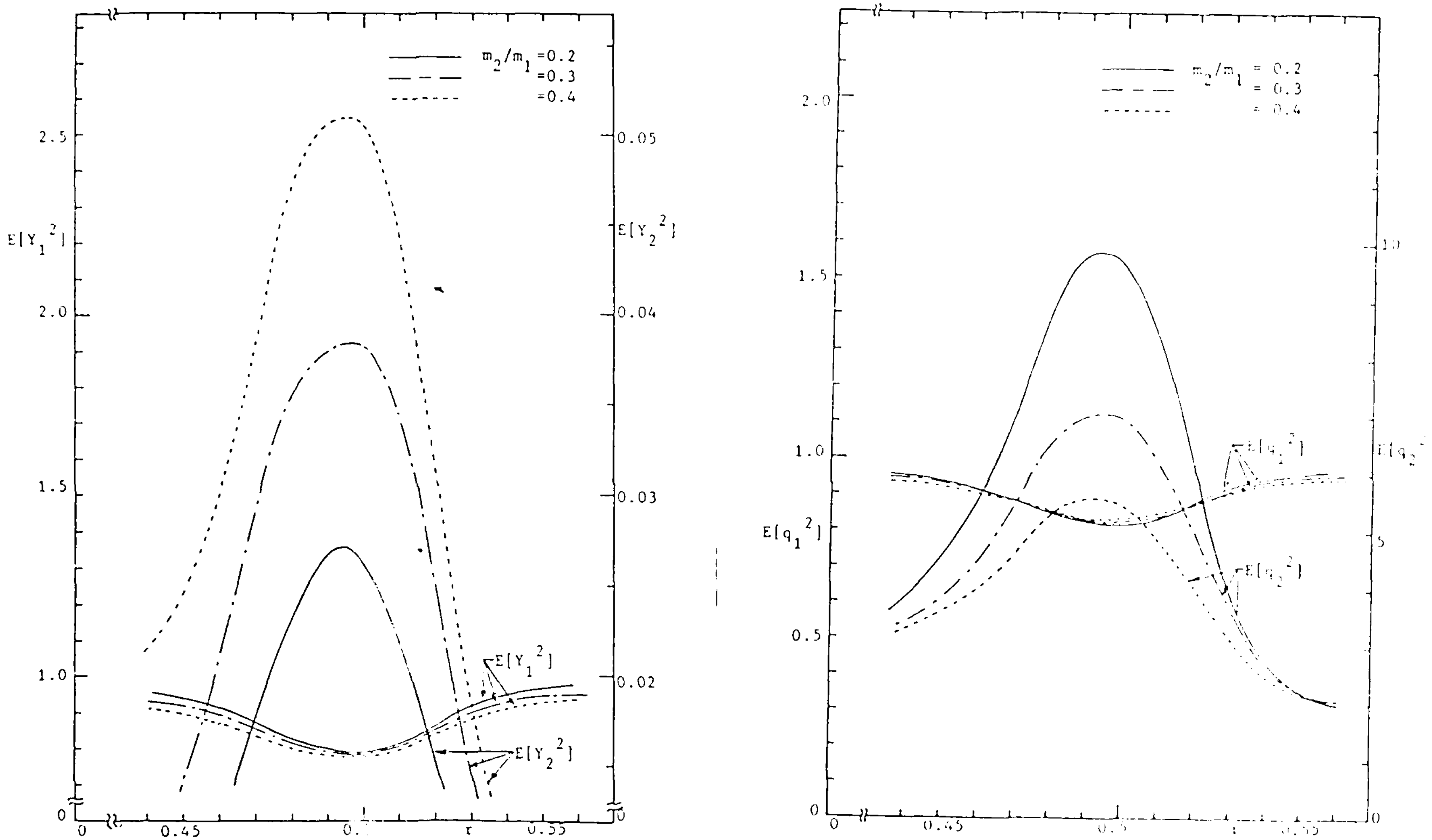
Fig. III.7 Non-Gaussian closure solution for various values of damping ratio.



(a) Normal mode mean square response.

(b) Generalized mean square response.

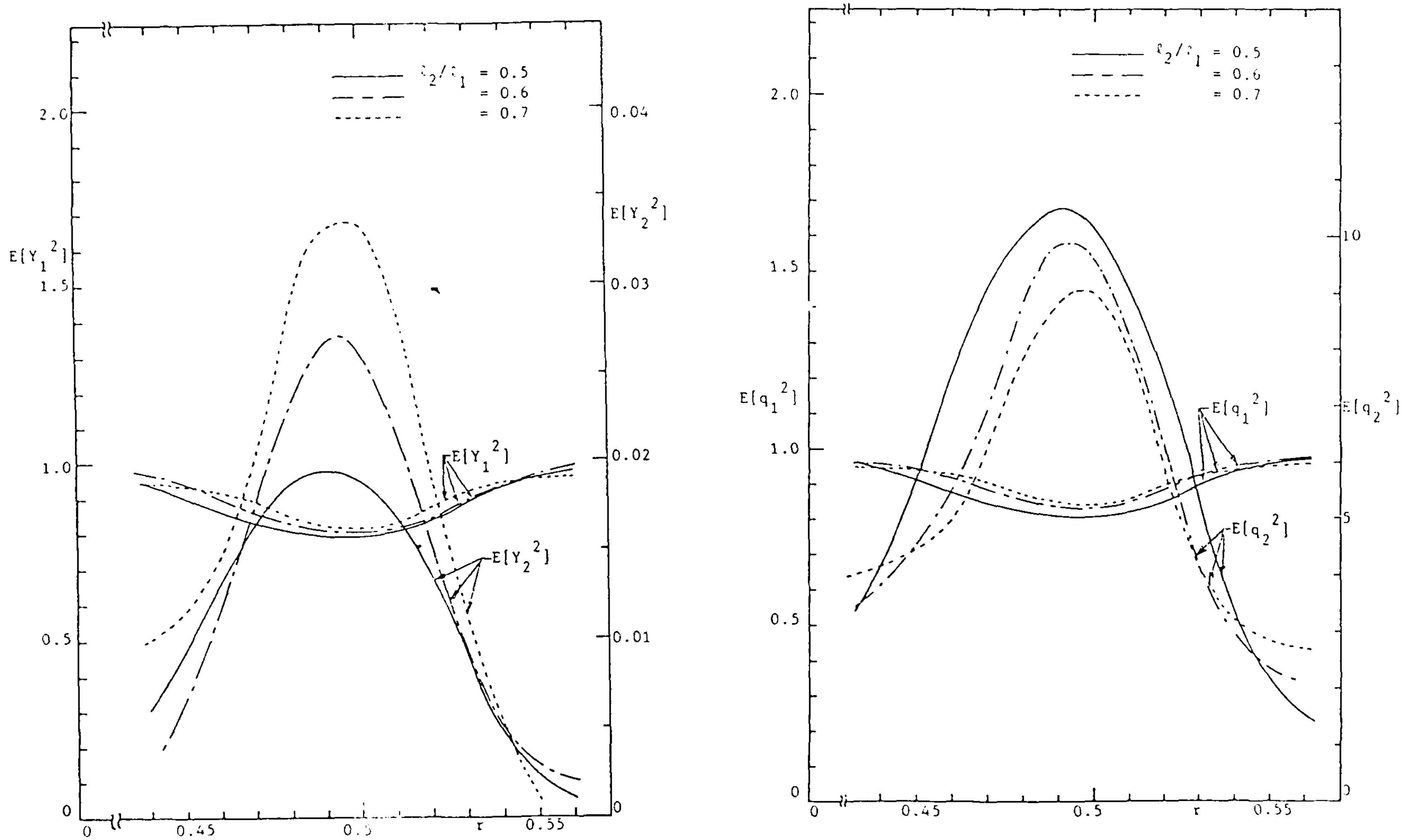
Fig. III.8 Non-Gaussian closure solution for various values of non-linear coupling parameter ε.



(a) Normal mode mean square response.

(b) Generalized mean square response.

Fig. III.9 Non-Gaussian closure solution for various values of mass ratio.



(a) Normal mode mean square response.

(b) Generalized mean square response.

Fig. III.10 Non-Gaussian closure solution for various values of length ratio.

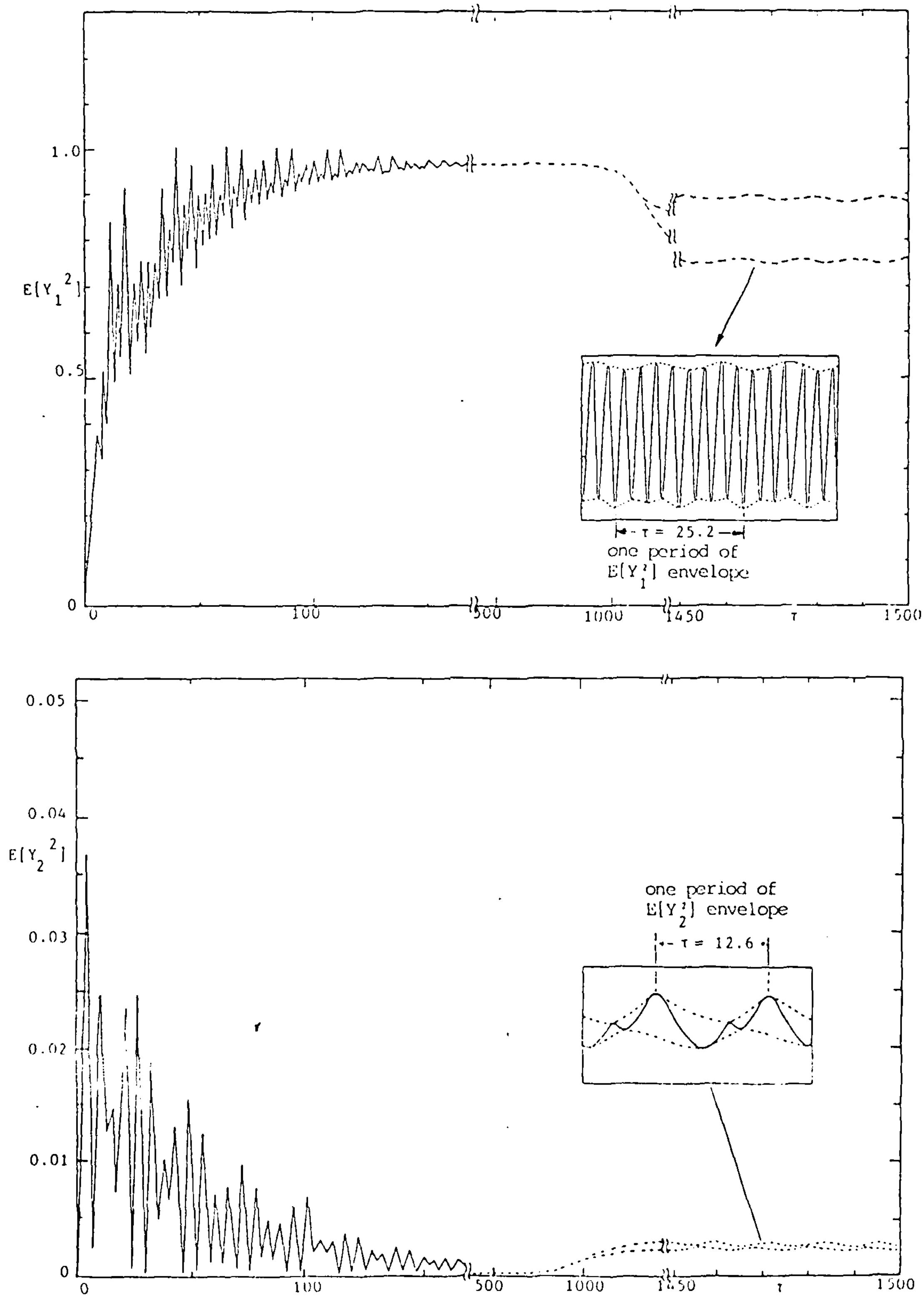


Fig. III.11 Time history of Gaussian response with initial conditions $E[Y_1Y_2Y_1'Y_2'] = 0.00001$ and the rest $= 0.0$.

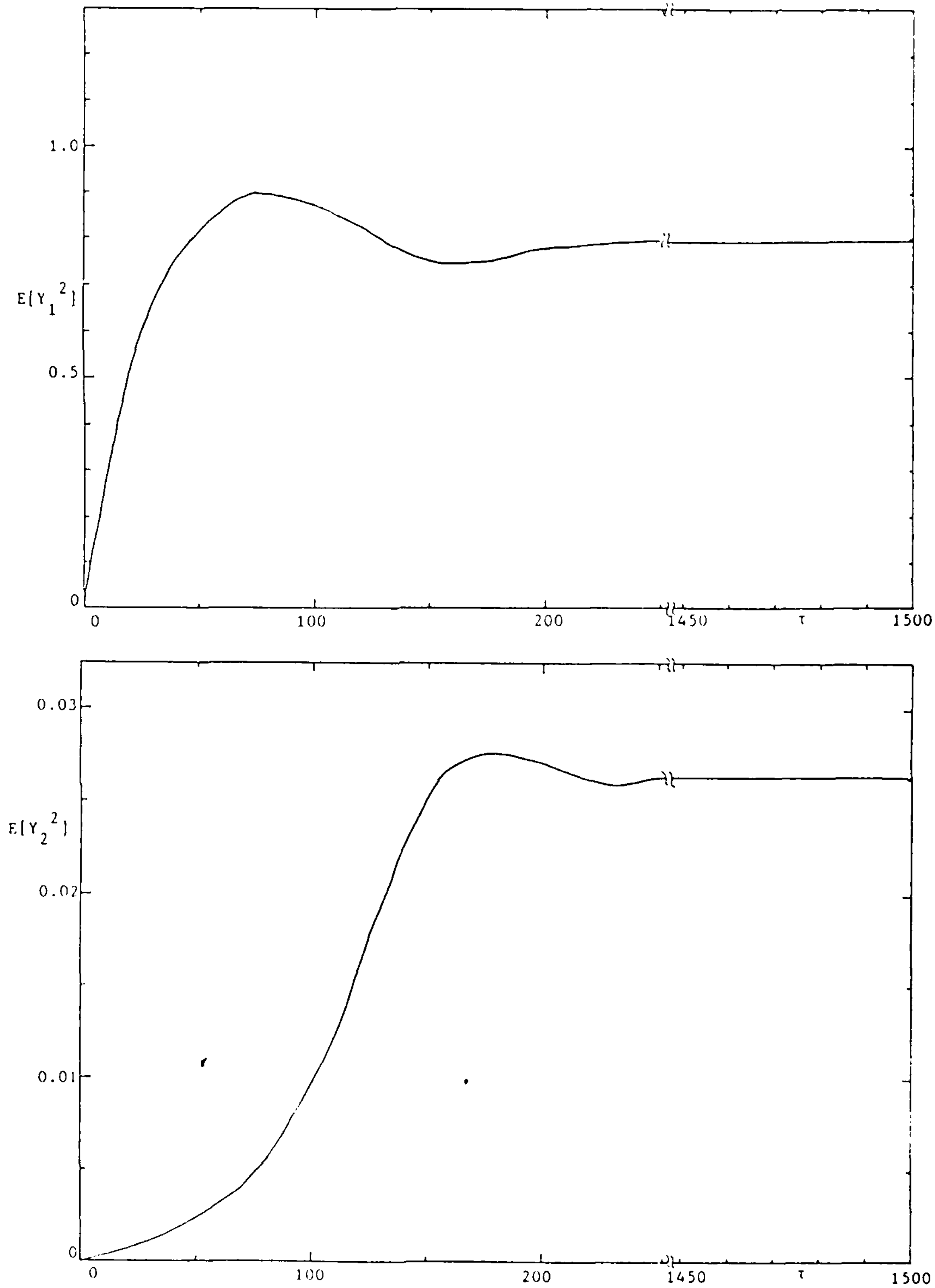


Fig. III.12 Time history of non-Gaussian response with initial conditions $E[Y_1 Y_2 Y_1' Y_2'] = 0.00001$ and the rest $= 0.0$.

III.5 Conclusions

The random response of an aeroelastic structure with autoparametric coupling has been investigated in the neighborhood of the internal resonance condition $r = \omega_2/\omega_1 = 0.5$. The response has been determined via two closure schemes: Gaussian and non-Gaussian. In the time domain, the transient response in both schemes exhibits the well-known characteristics of autoparametric interaction in a form of energy exchange between the displacement mean squares of the two modes. Furthermore, the level of these mean squares may exceed the mean squares during the steady state period. The Gaussian closure solution yields a quasi-stationary response while the non-Gaussian closure solution gives a stationary response.

The stationarity of the non-Gaussian closure solution may be confirmed from the analytical solution of Schmidt [9] who employed the stochastic averaging method to a non-linear two-degree-of-freedom system. However, Schmidt could not derive a closed form expression for the normalized constant of integration of the response probability density. The present investigation gives the dynamicist some guidelines for using the Gaussian or non-Gaussian closures in more complicated systems where none of the analytical approaches is applicable.

CHAPTER IV

NON-LINEAR RANDOM RESPONSE IN THE PRESENCE OF PARAMETER UNCERTAINTY

IV.1 Introduction

The dynamic behavior of an aeroelastic non-linear two-degree-of-freedom system has been examined in Chapter III. The random excitation appeared in the equations of motion as a non-homogeneous term and as a parametric coefficient. It was assumed that both damping and stiffness properties of the structure were time independent (constants). As the structure oscillates these properties may experience a certain degree of fluctuation as a result of the inherent temperature variation. Measurements taken in laboratory experiments often show that dynamic properties such as damping and stiffness of vibratory systems are non-repeatable parameters; every experiment gives different values for damping and stiffness of the same system, and the differences are random.

The dynamic response of structures with randomly varying parameters is of great practical interest to the analyst or designer using lumped parameter models, because the random variation of some system parameters may cause the system respone to exceed design specifications. These random uncertainties can be classified into two main categories [10]: statistical and non-statistical. To cite just one example, statistical uncertainties can be due to the stiffness or damping fluctuations caused by random variation in material properties or variation caused by manufacturing and assembly techniques. Non-statistical uncertainties can be due to the approximation involved in the mathematical modeling of complex structural systems.

It is very important to distinguish between two different problems encountered in mechanical vibrations and aeroelastic flutter. These are the random response of dynamic systems to random parametric excitations which appear as coefficients in the equations of motion, and the random response to random external excitations when the system parameters are not precisely defined and represented in a probabilistic sense. In the former case the system equations of motion are stochastic differential equations with time random coefficients. In the latter case the equations of motion are differential equations with random constant coefficients [11] or with coefficients that vary randomly with the spatial coordinates (random fields). Systems with parameter uncertainties are referred to in the literature as "disordered systems." The methods of treating dynamic systems under parametric random excitations are different from those used in solving differential equations with random constant coefficients or random fields.

In problems involving random constant coefficients the engineer is concerned with three main problems: random eigenvalues, random response, and optimum design. With reference to linear disordered systems these three problems will be reviewed in the next two sections.

IV.2 Random Eigenvalues

The eigenvalue of simple single degree-of-freedom systems is given by the square root of the stiffness to mass ratio. This value is assumed constant for identical systems. However, the actual eigenvalue of each system deviates from the original calculated one because in reality the physical properties of the elements can neither be measured exactly nor

manufactured exactly. Thus, the eigenvalues are random variables whose statistical properties are determined by the random coefficients of the inertia and stiffness terms of the equations of motion. In this case one may be interested in determining the probability that one or more eigenvalues lie in a given range or less than a certain value. Alternatively, one may need to know the probability that the smallest eigenvalue is specified in a given range [12]. Boyce and Goodwin [13] classified parameter uncertainties into two classes. These are:

(i) uncertainties in the geometry and the material properties, and

(ii) uncertainties in the support mechanism of the system.

These uncertainties appear in the equations of motion or in the boundary conditions, respectively. Under these types of uncertainties the eigenvalue has been determined for a limited class of dynamic systems. Boyce [12] addressed a number of techniques to determine the statistics of the eigenvalues of systems described by partial differential equations and boundary conditions involving uncertainty in their parameters. Two mathematical approaches known as "honest" and "dishonest" have been adopted in the literature [14]. In the honest approach the eigenvalues are first expressed in terms of the system parameters. The statistical characteristics of this solution are then determined in terms of the statistical characteristics of the random parameters. However, this approach involves difficulties since it is not possible to express the eigenvalue exactly except for very few simple cases. Four honest methods are outlined by Boyce [12]. These are the variational principles, perturbation methods, the kernel trace estimates, and asymptotic estimates. In the dishonest approach the statistical moments of the eigenvalues are directly

determined by performing averaging analysis to the system partial differential equations and its associated boundary conditions.

In a series of papers, Purkert and Scheidt [15-17] established a number of theorems pertaining to functionals of weakly correlated processes. These processes are encountered in the eigenvalue problems, boundary value problems and initial value problems. Purkert and Scheidt treated the stochastic eigenvalue problem for ordinary differential equations with deterministic boundary conditions. The coefficients of the differential operator were assumed to be independent weakly correlated processes of small correlation length. As the correlation length vanishes the eigenvalues and eigenvectors were found to possess Gaussian distributions. In their recent monograph, Scheidt and Purkert [18] treated the moments of the eigenvalues and mode shapes of random matrices and random ordinary differential operators. The calculations of these moments were based on perturbation expansions, and so require the random terms to be appropriately small.

Soong and Bogdanoff [19] examined the statistical properties of the natural frequencies of a linear n-degree-of-freedom system whose properties are known in a stochastic sense. They used a method based on the transfer matrix, developed originally by Kerner [20], together with a perturbation type expansion. For a linear 10 degrees-of-freedom system with random parameters having normal distribution with small standard deviation it was found that the top few natural frequencies have values which are very sensitive to the parameter variations, whereas the lowest few are insensitive to these variations. They derived explicit expressions for the natural

frequencies in terms of the parameter variations. These expressions can be used in estimating the changes in frequencies produced by deterministic parameter changes. Bliven and Soong [21] determined the statistics of the natural frequencies of a simply supported elastic beam with random imperfections in the beam stiffness. The beam was modeled as a lumped-parameter model and the same technique of Soong and Bogdanoff was employed. Bliven and Soong found that when the stiffness fluctuation has zero correlation distance the natural frequency standard deviation vanishes. The standard deviation reaches the value of 0.5 when stiffness variation is perfectly correlated. In addition, the standard deviation of the beam natural frequency was found to be insensitive to the number of segments in the lumped parameter model.

Collins and Thomson [22] treated the problem of eigenvalue and eigenvector statistics of a simple chain of equal springs and masses with uncorrelated random masses or with random uncorrelated stiffnesses. They showed that the standard deviation of the frequency is governed linearly with the standard deviations of the masses and stiffnesses. This result was obtained earlier by Soong and Bogdanoff [19]. However, these linear relationships disappear when a correlation exists between the masses and stiffnesses.

Vaicaitis [23] employed a two-variable perturbation expansion procedure to determine the eigenvalues and normal modes of beams with random and/or non-uniform characteristics which do not deviate considerably from the beam mean properties. He used a Monte Carlo simulation to determine the statistical averages of beam eigenvalues and mode shapes. It was found that the eigenvalues and mode shapes deviate significantly from those of a

uniform beam. The difference was mainly attributed to the fact that gradual change in the beam stiffness was permitted. In this case the beam is "soft" at one end and "hard" at the other end.

IV.3 Random Response

In an attempt to examine certain aspects of the dynamic behavior of statistically defined systems, Bogdanoff and Cheanea [24] treated linear single degree-of-freedom systems with independent discrete distributions in the mass, damping, and stiffness coefficients. Small dispersion in the system parameters resulted in considerable dispersion in the system frequency response. Their analysis was based on a partial differential equation for the response joint density function. This equation is known as the Liouville equation [11] and is identical to the Fokker-Planck equation with zero diffusion coefficient. The impulse response of a single degree-of-freedom system with random parameters was determined by Chen and Soroka [25] by using a perturbation approach. They found that both the mean and standard deviation of the response were non-stationary and the standard deviation was 90 degrees out of phase from the mean. They concluded that for systems with a very high natural frequency, the uncertainty in the natural frequency has a very negligible effect on the response statistics. However, the effect is significant if the natural frequency is low. As the damping factor decreases, the dispersion from the mean becomes substantial. In another study, Chen and Soroka [26] considered the response of multi-degree-of-freedom systems. Their study indicated that the response statistics of disordered systems are higher than those of purely deterministic systems.

The instantaneous transient response statistics of an undamped linear multi-degree-of-freedom system, subjected to arbitrary but deterministic forcing functions, with stiffness uncertainty was investigated by Prasthofer and Beadle [10]. For the case of an impulsive excitation to a single degree-of-freedom system, they found that the growth of the response uncertainty is exponential. As the standard deviation of the stiffness increases the response mean square increases rapidly with time. For a multi-degree-of-freedom system the response decay rate decreases as the correlation coefficient between the stiffness elements increases.

The influence of damping uncertainty on the frequency response of a linear multi-degree-of-freedom system was examined by Caravani and Thomson [27]. They determined the mean and standard deviation of the response by using a linearization technique and a Monte Carlo simulation. They pointed out that an accurate estimate of the damping coefficients for lightly damped systems, in the neighborhood of a natural frequency, is very important in determining the mean and standard deviation of the system response.

IV.4 Design Optimization

During the design stage of structural systems the fluctuations of their dynamic characteristics such as response or eigenvalues should be defined. The main problem is how to restrict the fluctuations of the system parameters. For example, in systems in which the values of displacement are significant, or in structures for which the safety factors for fatigue strength are determined in terms of probability functions, the problem is to set up an optimum standard of manufacturing their components. Here the permissible fluctuation in the characteristics becomes a restrictive

condition. Under these circumstances, the designer is encountered with a problem of optimization to specify the maximum permissible fluctuations of parameters.

Tanaka and Onishi [28] developed a method of regulating the deviations of random parameters and derived a restrictive conditional formula from the permissible fluctuation of displacements or natural frequencies. The method is based on the linear deviation analysis with partial differential analysis together with sequential linear programming (SLP) for a number of restrictive conditions. Tanaka, et al. [29] treated the optimization problem of the allowable variance of random parameters by using a perturbation method and Monte Carlo simulation. They computed the deviation of the steady state response of structural systems with random parameters with the purpose of regulating the deviation of the random parameters when the deviation of the response is specified. Rao [30] applied the multi-objective optimization techniques to the design of simple structures involving uncertain parameters and stochastic processes. The necessity of optimizing the structural systems involving dynamic restrictions, random parameters, stochastic processes, and multiple objectives has been outlined by Rao [31].

In this chapter, the effects of randomly time-varying damping and stiffness (represented by stochastic processes) on the system response will be investigated by using the Fokker-Planck equation approach. Gaussian and non-Gaussian closure schemes will be used to obtain the mean square response of the system in the normal and generalized coordinates. The present analysis will not provide any information about the stochastic stability of the system. The investigation of stochastic stability of aeroelastic structures will be examined in another report.

IV.5 Theoretical Analysis

Considering again the aeroelastic system shown in Fig. I.1, and allowing random variations in the damping and stiffness coefficients, equations (III.3) become

$$
\begin{aligned}
& Y_1'' + 2\zeta_1[1 + \xi_{c_1}(\tau)]Y_1' + [1 + \xi_{k_1}(\tau)]Y_1 = -\xi''(\tau)a_1 \\
& \quad - \xi''[a_2Y_1 + a_3Y_2] + a_4Y_1Y_1'' + a_5Y_1Y_2'' + a_6Y_2Y_1'' + a_7Y_2Y_2'' + a_8Y_1'^2 \\
& \quad + a_9Y_1'Y_2' + a_{10}Y_2'^2 \\
& Y_2'' + 2\zeta_2 r[1 + \xi_{c_2}(\tau)]Y_2' + r^2[1 + \xi_{k_2}(\tau)]Y_2 = -\xi''(\tau)b_1 \\
& \quad - \xi''[b_2Y_1 + b_3Y_2] + b_4Y_1Y_1'' + b_5Y_1Y_2'' + b_6Y_2Y_1'' + b_7Y_2Y_2'' \\
& \quad + b_8Y_1'^2 + b_9Y_1'Y_2' + b_{10}Y_2'^2
\end{aligned}
\tag{IV.1}
$$

where $\xi_{c_i}(\tau)$ and $\xi_{k_i}(\tau)$ are assumed independent Gaussian random processes in damping and stiffness in the ith mode, respectively, (i = 1, 2). Equations (IV.1) can be approximated through successive elimination of Y_i'' from non-linear terms and will be transformed into the Stratonovich type equation [1]

$$X_i' = f_i(\underset{\sim}{X},\tau) + \sum_j G_{ij}(\underset{\sim}{X},\tau)\xi_i(\tau) \tag{IV.2}$$

through the coordinate transformation

$$\begin{Bmatrix} Y_1 \\ Y_2 \\ Y_1' \\ Y_2' \end{Bmatrix} = \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{Bmatrix} \tag{IV.3}$$

equations (IV.1) can be written in the form

$$
\begin{aligned}
X_1' &= X_3 \\
X_2' &= X_4 \\
X_3' &= -X_1 - 2\zeta_1 X_3 - a_4 X_1^2 - (a_6 + r^2 a_5) X_1 X_2 - r^2 a_7 X_2^2 - 2\zeta_1 a_4 X_1 X_3 \\
&\quad - 2\zeta_2 r a_5 X_1 X_4 - 2\zeta_1 a_6 X_2 X_3 - 2\zeta_2 r a_7 X_2 X_4 + a_8 X_3^2 + a_9 X_3 X_4 + a_{10} X_4^2 \\
&\quad - X_1 \xi_{k_1}(\tau) - 2\zeta_1 [X_3 + a_4 X_1 X_3 + a_6 X_2 X_3] \xi_{c_1}(\tau) \\
&\quad - r^2 [a_5 X_1 X_2 + a_7 X_2^2] \xi_{k_2}(\tau) - 2\zeta_2 r [a_5 X_1 X_4 + a_7 X_2 X_4] \xi_{c_2}(\tau) \\
&\quad - [A_1 + A_2 X_1 + A_3 X_2 + A_4 X_1^2 + A_5 X_1 X_2 + A_6 X_2^2] \xi(\tau) \\
X_4' &= -r^2 X_2 - 2\zeta_2 r X_4 - b_4 X_1^2 - (b_6 + r^2 b_5) X_1 X_2 - r^2 b_7 X_2^2 \qquad \text{(IV.4)} \\
&\quad - 2\zeta_1 b_4 X_1 X_3 - 2\zeta_2 r b_5 X_1 X_4 - 2\zeta_1 b_6 X_2 X_3 - 2\zeta_2 r b_7 X_2 X_4 + b_8 X_3^2 \\
&\quad + b_9 X_3 X_4 + b_{10} X_4^2 - 2\zeta_1 (b_4 X_1 X_3 + b_6 X_2 X_3) \xi_{c_1}(\tau) \\
&\quad - r^2 [X_2 + b_5 X_1 X_2 + b_7 X_2^2] \xi_{k_2}(\tau) \\
&\quad - 2\zeta_2 r [X_4 + b_5 X_1 X_4 + b_7 X_2 X_4] \xi_{c_2}(\tau) \\
&\quad - [B_1 + B_2 X_1 + B_3 X_2 + B_4 X_1^2 + B_5 X_1 X_2 + B_6 X_2^2] \xi(\tau)
\end{aligned}
$$

Alternatively, equations (IV.4) can be written in the Itô form [1]

$$
dX_i = \left[f_i(\underset{\sim}{X},\tau) + \frac{1}{2} \sum_k \sum_j G_{kj} \frac{\partial G_{ij}}{\partial X_k} \right] d\tau + \sum_j G_{ij}(\underset{\sim}{X},\tau) dB_i(\tau) \qquad \text{(IV.5)}
$$

where the random processes $\xi_i(\tau)$ have been replaced by white noise processes $W_i(\tau)$. In (IV.5) the formal derivative of the Brownian motion $W_i(\tau) = \frac{dB_i(\tau)}{d\tau}$ is used. $dB_i(\tau)$ are independent Brownian motion processes with properties

$$
\begin{aligned}
E[dB_i(\tau)] &= 0 \\
E[dB_i^2(\tau)] &= 2D_i d\tau = \sigma_i^2 d\tau
\end{aligned}
\tag{IV.6}
$$

Introducing the Itô correction term represented by the double summation expression in equation (IV.5), equatons (IV.4) become

$$
\begin{aligned}
dX_1 &= X_3 d\tau \\
dX_2 &= X_4 d\tau \\
dX_3 &= \{- X_1 - 2\zeta_1 X_3 - a_4 X_1^2 - (a_6 + r^2 a_5) X_1 X_2 - r^2 a_7 X_2^2 - 2\zeta_1 a_4 X_1 X_3 \\
&\quad - 2\zeta_2 r a_5 X_1 X_4 - 2\zeta_1 a_6 X_2 X_3 - 2\zeta_2 r a_7 X_2 X_4 + a_8 X_3^2 + a_9 X_3 X_4 + a_{10} X_4^2 \\
&\quad + 4D_{c_2} \zeta_2^2 r^2 [a_5 X_1 X_4 + a_7 X_2 X_4 + a_5 b_5 X_1^2 X_4 + (a_5 b_7 + a_7 b_5) X_1 X_2 X_4 \\
&\quad + a_7 b_7 X_2^2 X_4]\} d\tau - X_1 dB_{k_1}(\tau) - r^2 [a_5 X_1 X_2 + a_7 X_2 X_4] dB_{k_2}(\tau) \\
&\quad - 2\zeta_1 [X_3 + a_4 X_1 X_3 + a_6 X_2 X_3] dB_{c_1}(\tau) \\
&\quad - 2\zeta_2 r [a_5 X_1 X_4 + a_7 X_2 X_4] dB_{c_2}(\tau) \\
&\quad - [A_1 + A_2 X_1 + A_3 X_2 + A_4 X_1^2 + A_5 X_1 X_2 + A_6 X_2^2] dB(\tau)
\end{aligned}
\tag{IV.7}
$$

$$dX_4 = \{- r^2X_2 - 2\zeta_2 rX_4 - b_4X_1^2 - (b_6 + r^2b_5)X_1X_2 - r^2b_7X_2^2$$

$$- 2\zeta_1 b_4X_1X_3 - 2\zeta_2 rb_5X_1X_4 - 2\zeta_1 b_6X_2X_3 - 2\zeta_2 rb_7X_2X_4$$

$$+ b_8X_3^2 + b_9X_3X_4 + b_{10}X_4 + 4D_{c_1}\zeta_1^2[b_4X_1X_3 + b_6X_2X_3$$

$$+ a_4b_4X_1^2X_3 + (a_4b_6 + b_4a_6)X_1X_2X_3 + a_6b_6X_2^2X_3]$$

$$+ 4D_{c_2}\zeta_2^2r^2[X_4 + 2b_5X_1X_4 + 2b_7X_2X_4 + b_5^2X_1^2X_4 + 2b_5b_7X_1X_2X_4$$

$$+ b_7^2X_2^2X_4]\}d\tau - 2\zeta_1[b_4X_1X_3 + b_6X_2X_3]dB_{c_1}(\tau)$$

$$- r^2[X_2 + b_5X_1X_2 + b_7X_2^2]dB_{k_2}(\tau)$$

$$- 2\zeta_2 r[X_4 + b_5X_1X_4 + b_7X_2X_4]dB_{c_2}(\tau)$$

$$- [B_1 + B_2X_1 + B_3X_2 + B_4X_1^2 + B_5X_1X_2 + B_6X_2^2]dB(\tau) \qquad \text{(IV.7) cont'd.}$$

The general differential equation of the response joint moments is obtained by using the Fokker-Planck equation approach as outlined in Chapter III. This procedure results in the moment equation:

$$m'_{i,j,k,\ell} = im_{i-1,j,k+1,\ell} + jm_{i,j-1,k,\ell+1}$$

$$+ k\{- m_{i+1,j,k-1,\ell} + \zeta_1(\zeta_1D_{c_1} - 2)m_{i,j,k,\ell} - a_4m_{i+2,j,k-1,\ell}$$

$$- (a_6 + r^2a_5)m_{i+1,j+1,k-1,\ell} - r^2a_7m_{i,j+2,k-1,\ell}$$

$$+ 2\zeta_1a_4(\zeta_1D_{c_1} - 1)m_{i+1,j,k,\ell} + \zeta_2ra_5(\zeta_2rD_{c_2} - 2)m_{i+1,j,k-1,\ell+1}$$

$$+ 2\zeta_1a_6(\zeta_1D_{c_1} - 1)m_{i,j+1,k,\ell} + \zeta_2ra_7(\zeta_2rD_{c_2} - 2)m_{i,j+1,k-1,\ell+1}$$

$$+ a_8m_{i,j,k+1,\ell} + a_9m_{i,j,k,\ell+1} + a_{10}m_{i,j,k-1,\ell+2}\}$$

$$
\begin{aligned}
&+ \ell\{-r^2 m_{i,j+1,k,\ell-1} + \zeta_2 r(\zeta_2 r D_{c_2} - 2) m_{i,j,k,\ell} - b_4 m_{i+2,j,k,\ell-1} \\
&\quad - (r^2 b_5 + b_6) m_{i+1,j+1,k,\ell-1} - r^2 b_7 m_{i,j+2,k,\ell-1} \\
&\quad + \zeta_1 b_4(\zeta_1 D_{c_1} - 2) m_{i+1,j,k+1,\ell-1} + 2\zeta_2 r b_5(\zeta_2 r D_{c_2} - 1) m_{i+1,j,k,\ell} \\
&\quad + \zeta_1 b_6(\zeta_1 D_{c_1} - 2) m_{i,j+1,k+1,\ell-1} + 2\zeta_2 r b_7(\zeta_2 r D_{c_1} - 1) m_{i,j+1,k,\ell} \\
&\quad + b_8 m_{i,j,k+2,\ell-1} + b_9 m_{i,j,k+1,\ell} + b_{10} m_{i,j,k,\ell+1}\} \\
&+ k(k-1)\{DA_1^2 m_{i,j,k-2,\ell} + 2DA_1A_2 m_{i+1,j,k-2,\ell} \\
&\quad + 2DA_1A_3 m_{i,j+1,k-2,\ell} + [D_{k_1} - D(2A_1A_4 + A_2^2)] m_{i+2,j,k-2,\ell} \\
&\quad + 2D(A_1A_5 + A_2A_3) m_{i+1,j+1,k-2,\ell} + D(A_3^2 + 2A_1A_6) m_{i,j+2,k-2,\ell} \\
&\quad + 4D_{c_1} \zeta_1^2 m_{i,j,k,\ell}\} \qquad\qquad \text{(IV.8)} \\
&+ k\ell\{2DA_1B_1 m_{i,j,k+1,\ell-1} + 2D(A_1B_2 + A_2B_1) m_{i+1,j,k-1,\ell-1} \\
&\quad + 2D(A_1B_3 + A_3B_1) m_{i,j+1,k-1,\ell-1} \\
&\quad + 2D(A_1B_4 + A_4B_1 + A_2B_2) m_{i+2,j,k-1,\ell-1} \\
&\quad + 2D(A_1B_5 + A_5B_1 + A_2B_3 + A_3B_2) m_{i+1,j+1,k-1,\ell-1} \\
&\quad + 2D(A_3B_3 + A_1B_6 + A_6B_1) m_{i,j+2,k-1,\ell-1}\} \\
&+ \ell(\ell-1)\{2DB_1^2 m_{i,j,k,\ell-2} + 2DB_1B_2 m_{i+1,j,k,\ell-2} \\
&\quad + 2DB_1B_3 m_{i,j+1,k,\ell-2} + D(2B_1B_4 + B_2^2) m_{i+2,j,k,\ell-2} \\
&\quad + 2D(B_1B_5 + B_1B_3) m_{i+1,j+1,k,\ell-2} \\
&\quad + [D_{k_2} r^2 + D(B_3^2 + 2B_1B_6)] m_{i,j+2,k,\ell-2} + 4D_{c_2}\zeta_2^2 r^2 m_{i,j,k,\ell}\}
\end{aligned}
$$

Again, this equation reveals that the response moment equations constitute an infinite hierarchy set. This infinite hierarchy may be closed via Gaussian or non-Gaussian schemes.

For the Gaussian closure scheme, fourteen equations of first order and second order moments will be generated from equation (IV.8). These equations are coupled with third order moments, and if the third order cumulant is set to zero, the fourteen equations can be closed. The response of the system can then be determined by integrating the fourteen moment equations using the IMSL DVERK routine.

For the non-Gaussian solution, which is more accurate, 69 moment equations for the first four orders are generated. These equations are found to be coupled through fifth order moment terms which can be closed by setting the corresponding cumulant to zero. The closed 69 moment equations are integrated numerically by again using the IMSL DVERK routine.

IV.6 Response of the System with Damping Uncertainty

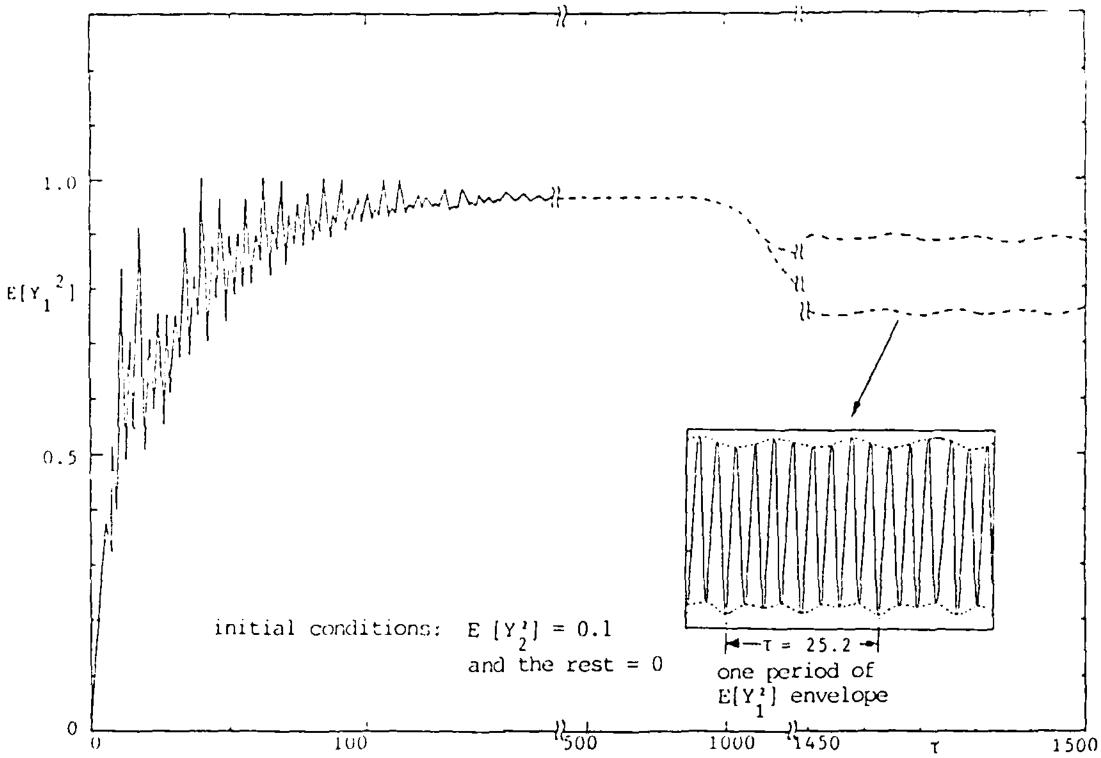
The time history response according to the Gaussian closure is plotted in Fig. IV.1 for internal tuning resonance $r = \omega_2/\omega_1 = 0.5$, damping ratios $\zeta_1 = \zeta_2 = 0.02$, mass ratio $m_2/m_1 = 0.2$, beam length ratio $\ell_2/\ell_1 = 0.6$, and spectral density of randomly varying damping $D_{c_1} = D_{c_2} = 0.1\ D$ where $2D = 0.08$ is the spectral density of the base motion. It is seen that after a long period of the time parameter $\tau = 1000$ the Gaussian time responses fluctuate between two limits, indicating that the system does not achieve a steady state.

For the initial conditions indicated on each figure, the first mode grows very fast with rapid fluctuations while the second mode shoots over its initial value with a general decay which reaches values below the initial value. The two modes exchange energy over the transient period $\tau = 150$. During the steady state period each mode fluctuates between two envelopes. The period of oscillation of the first mode envelope is twice that of the second mode. Another important feature is that the level of the mean squares during the transient period is higher than the steady-state level.

The time history response according to the non-Gaussian solution is shown in Fig. IV.2. This figure shows fluctuations during the transient response period $\tau = 160$. Contrary to the Gaussian solutions, the system response achieves a stationary state.

The effects of random variation in damping ratios ζ_1 and ζ_2 on the Gaussian and non-Gaussian autoparametric region in normal and generalized coordinates are shown in Figs. IV.3 and IV.4 as functions of the internal

tuning ratio $r = \omega_2/\omega_1$. In Gaussian closure, as damping is varying randomly, the autoparametric interaction region becomes more narrow and the peak values of the two mean square responses stay almost the same.



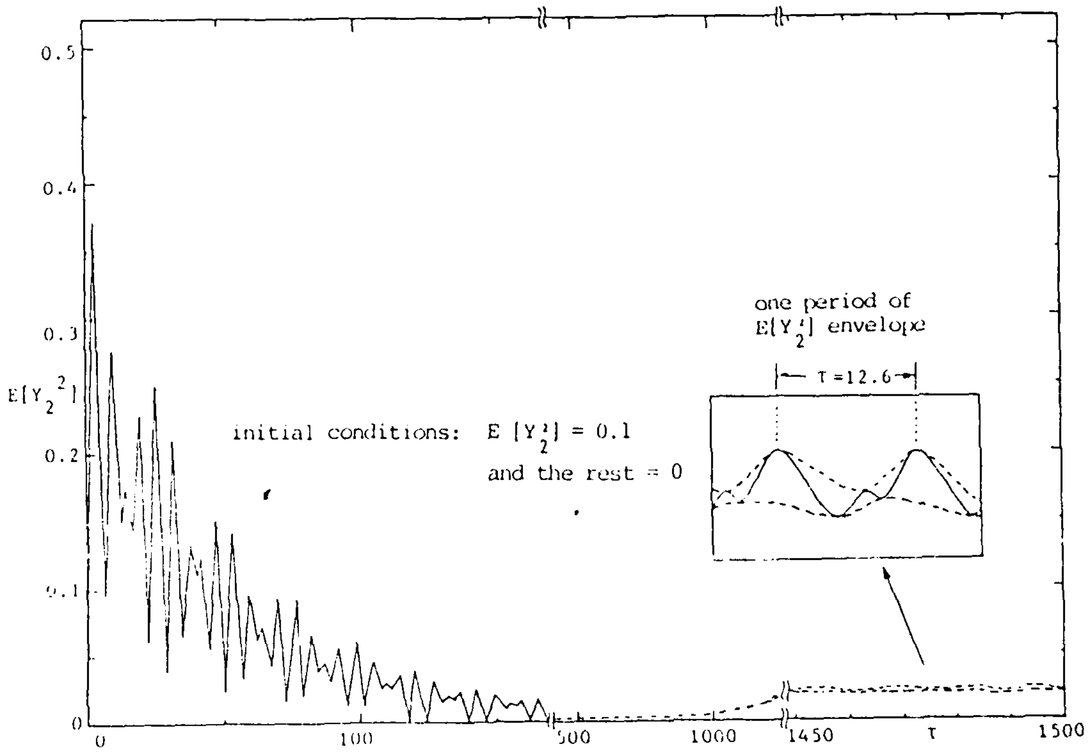


Fig. IV.1 Time history of Gaussian response for system with randomly varying damping ($D_{c_1} = D_{c_2} = 0.1$ D) and constant stiffness ($D_{k_1} = D_{k_2} = 0$).

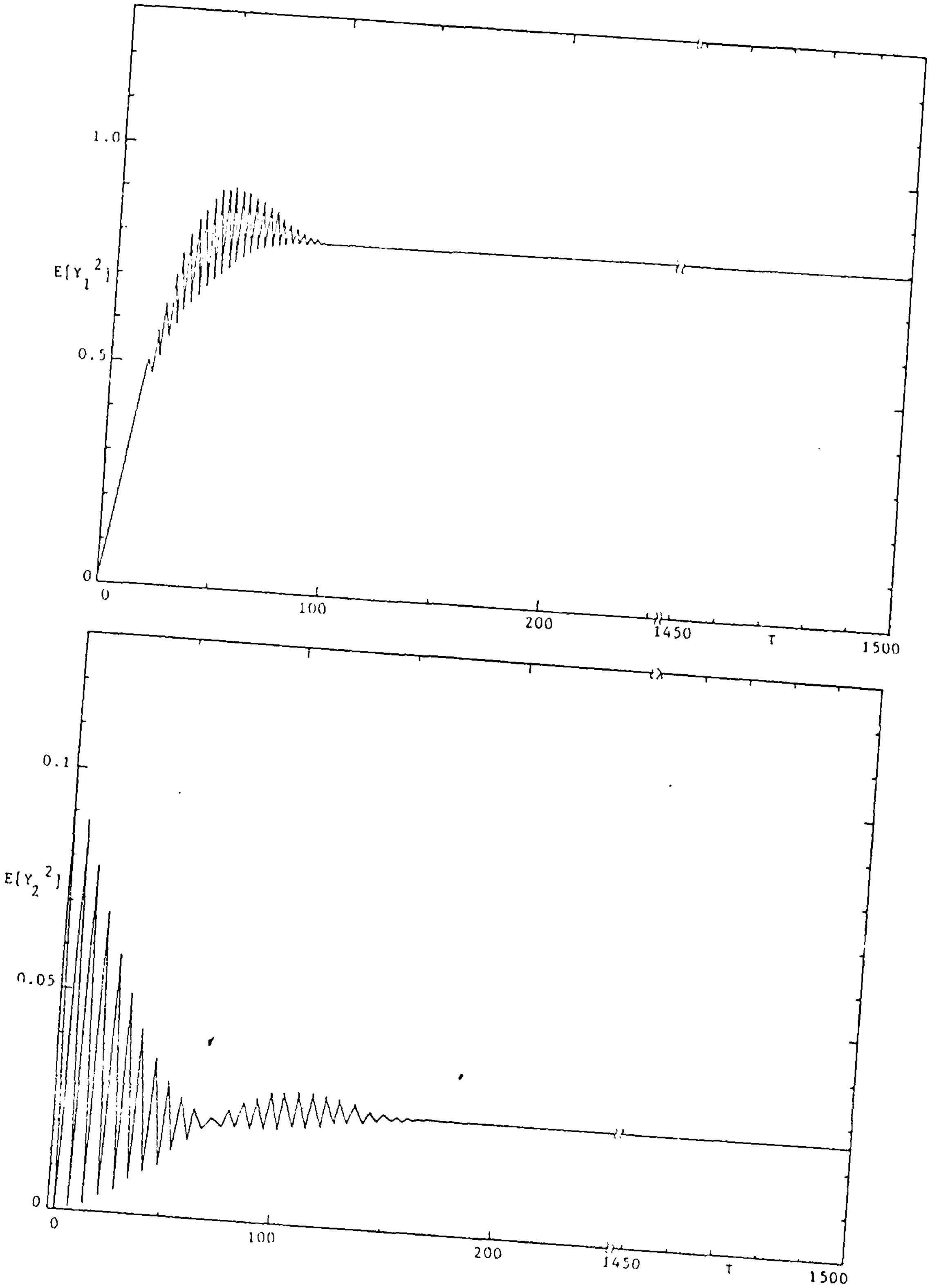
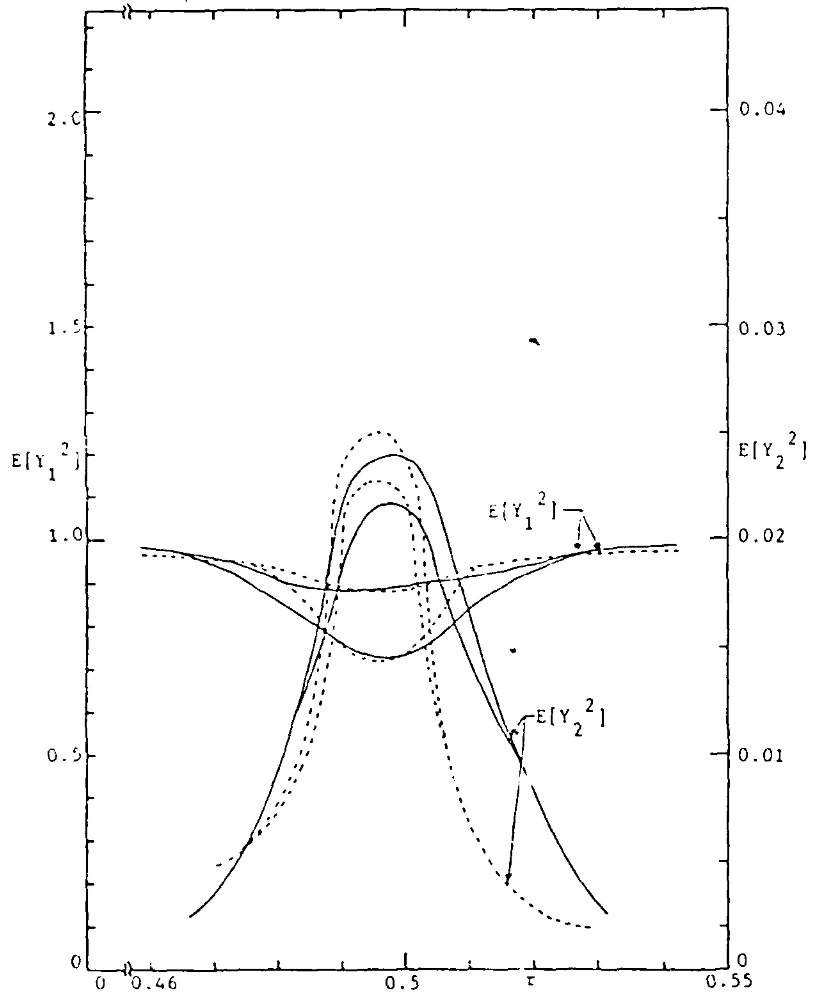
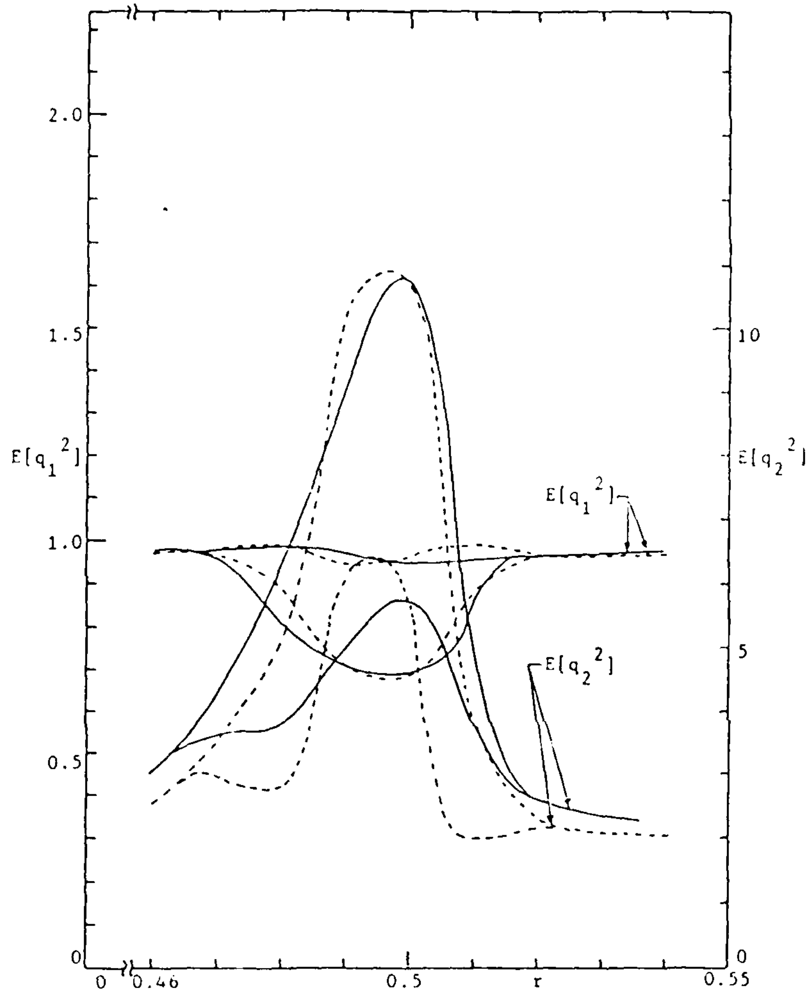


Fig. IV.2 Time history of non-Gaussian response of system with randomly varying damping ($D_{c_1} = D_{c_2} = 0.1$ D) and constant stiffness ($D_{k_1} = D_{k_2} = 0$).



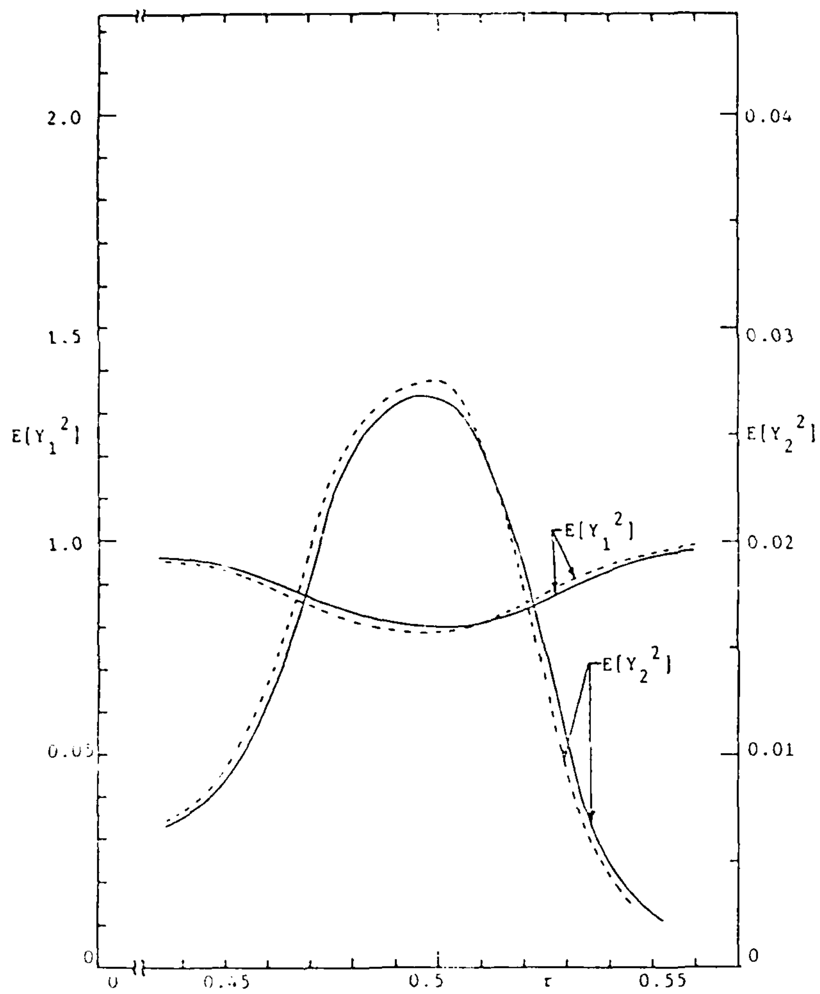
(a) Normal mode mean square response.



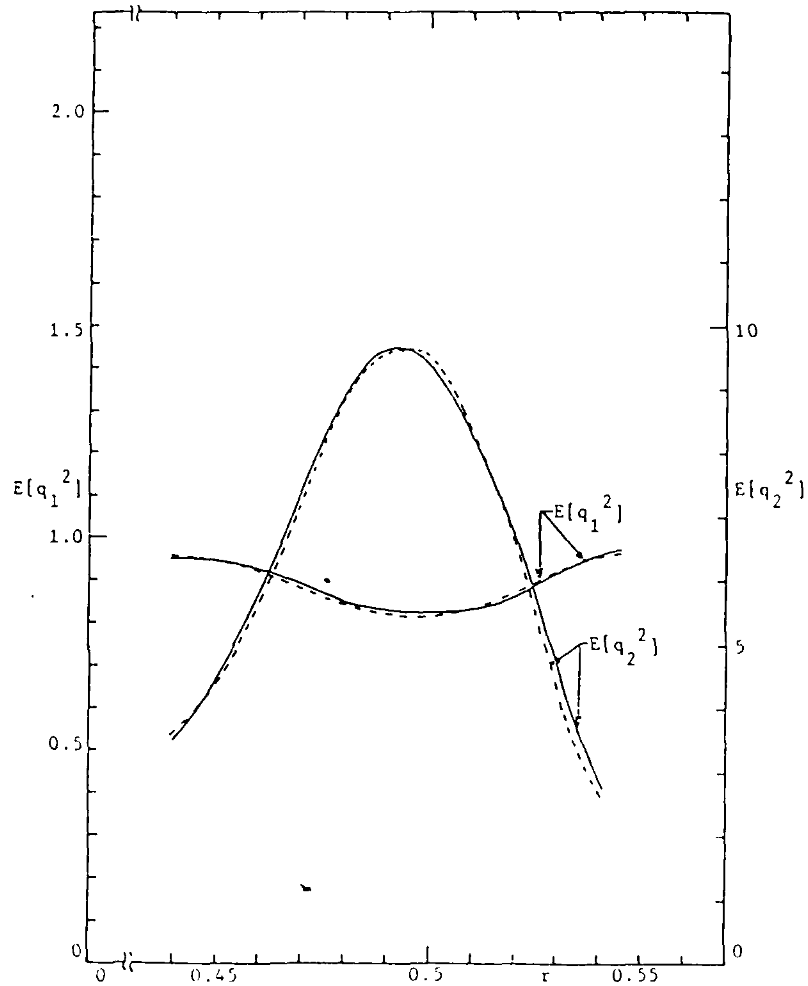
(b) Generalized mean square response.

Fig. IV.3 Gaussian closure solution for system with randomly varying damping.

__________ : constant damping and stiffness, - - - - - -: randomly varying damping ($D_{c_i} = 0.1\ D$, $D_{k_i} = 0$, $i = 1, 2$)



(a) Normal mode mean square response.



(b) Generalized mean square response.

Fig. IV.4 Non-Gaussian closure solution for system with randomly varying damping and constant stiffness.

———— : constant damping and stiffness, - - - - - -: randomly varying damping ($D_{c_i} = 0.1\ D$, $D_{k_i} = 0$, $i = 1, 2$)

IV.7 System Response with Stiffness Uncertainty

The system mean square responses are determined in this section when the stiffnesses of the system are varied randomly with spectral densities $D_{k_1} = D_{k_2} = 0.1\ D$, $(D_{c_1} = D_{c_2} = 0)$ where $2D = 0.08$ is the spectral density of the base motion. Figure IV.5 shows the time history responses according to the Gaussian closure scheme for internal resonance $r = 0.5$, damping ratios $\zeta_1 = \zeta_2 = 0.02$, mass ratio $m_2/m_1 = 0.2$, beam length ratio $\ell_2/\ell_1 = 0.6$, and non-linear coupling parameter $\varepsilon = 0.02$. The transient response exhibits the same features outlined in section IV.6 and the steady state response fluctuates between two quasi-stationary envelopes.

Figure IV.6 shows the time history responses according to the non-Gaussian closure scheme for the same system parameters as above. Again, the transient response fluctuates with energy exchange between two modes until the system achieves a stationary level.

Mean square responses obtained by Gaussian and non-Gaussian solutions are higher in the transient region and in the steady state than the corresponding values when the system possesses damping uncertainty only. Figures IV.7 and IV.8 demonstrate the system displacement mean squares as functions of the internal tuning parameter r, according to Gaussian and non-Gaussian solutions, respectively. It is seen that the inclusion of stiffness uncertainty has a remarkable effect on the mean square responses. The mean squares of the response displacements are relatively higher than those without stiffness uncertainty. In addition, although the region of autoparametric interaction is wider, the absorber effect is less pronounced.

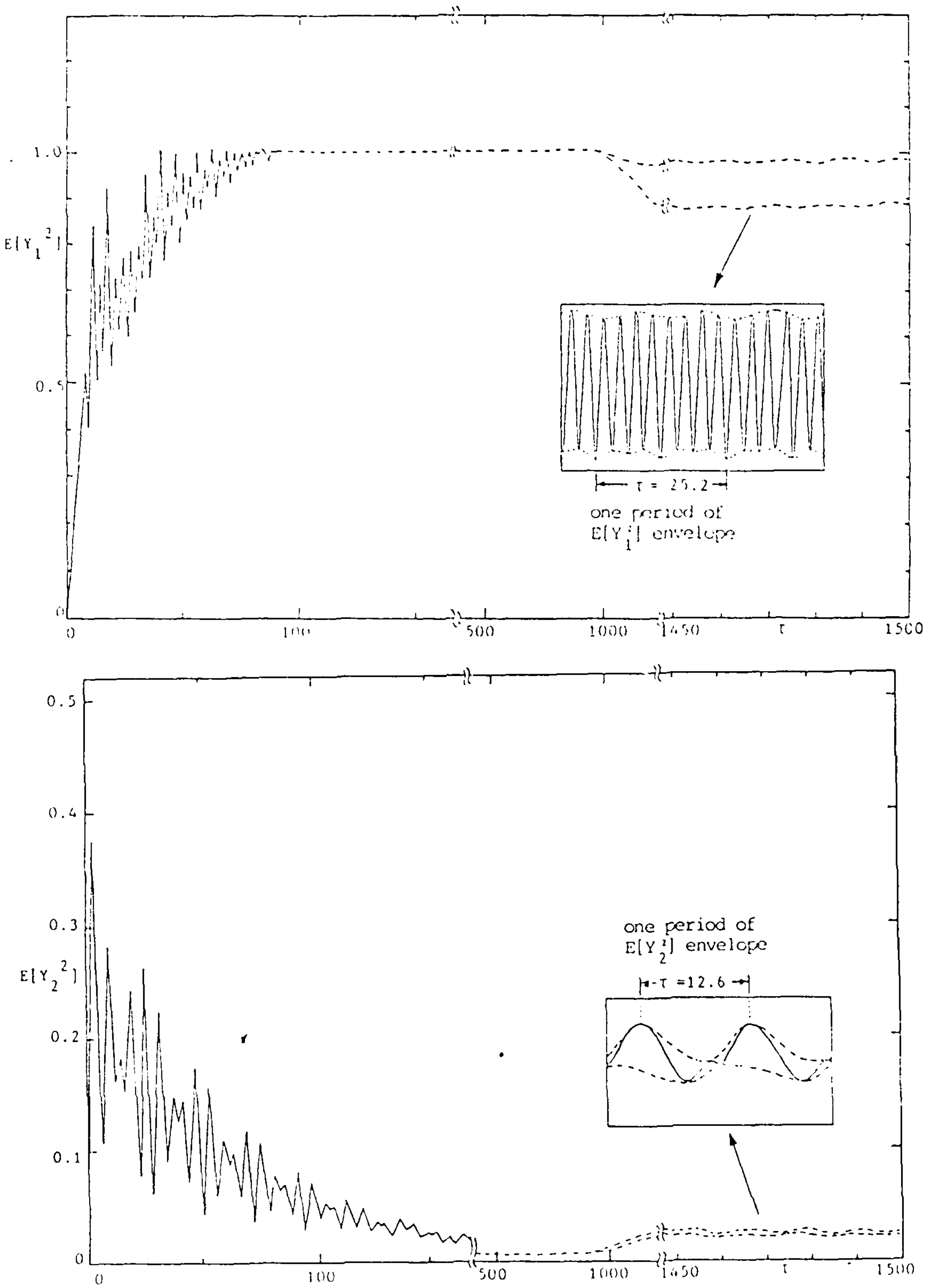


Fig. IV.5 Time history of Gaussian response of system with randomly varying damping ($D_{c_1} = D_{c_2} = 0.1$ D) and randomly varying stiffness ($D_{k_1} = D_{k_2} = 0.1$ D).

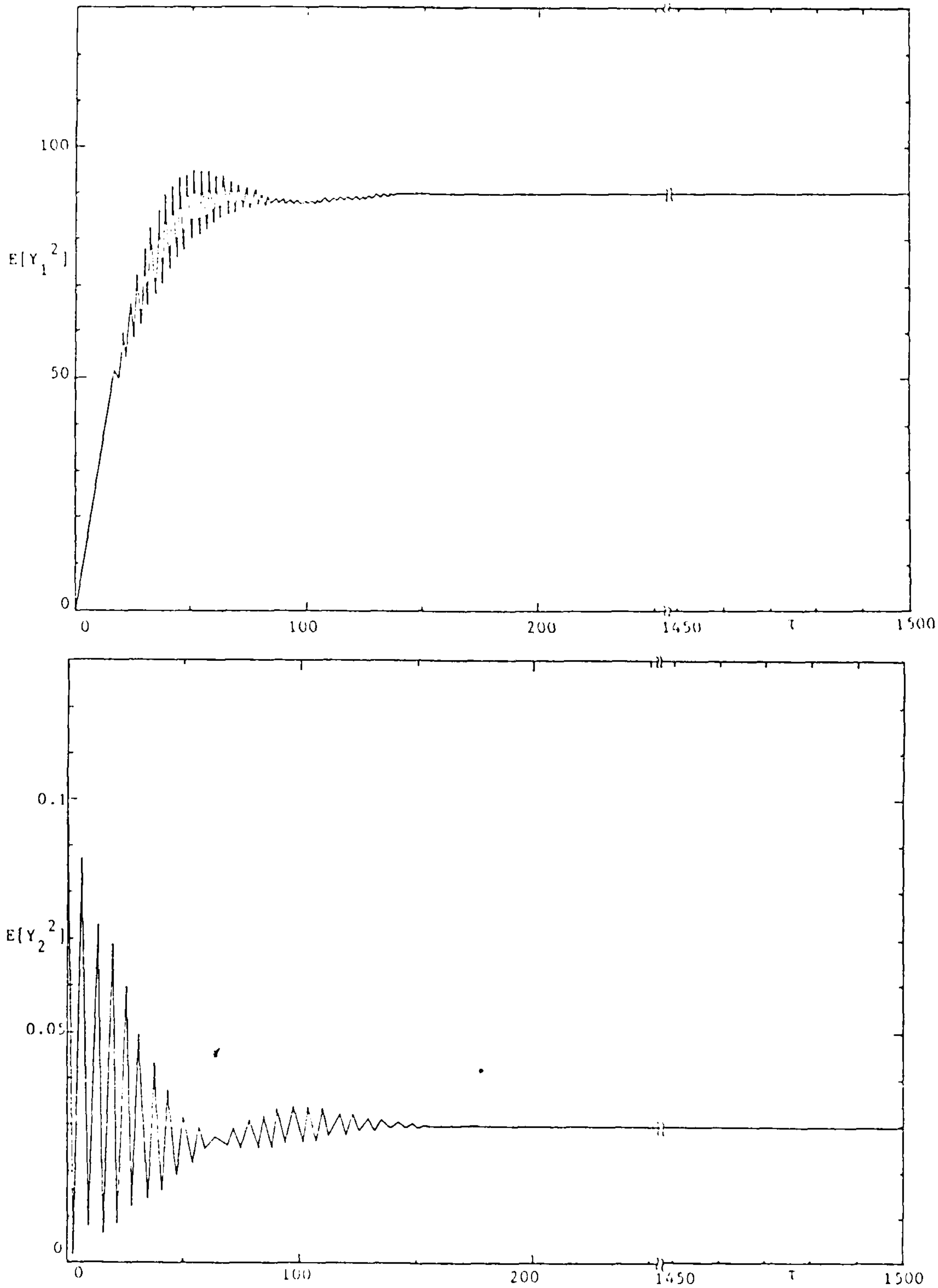
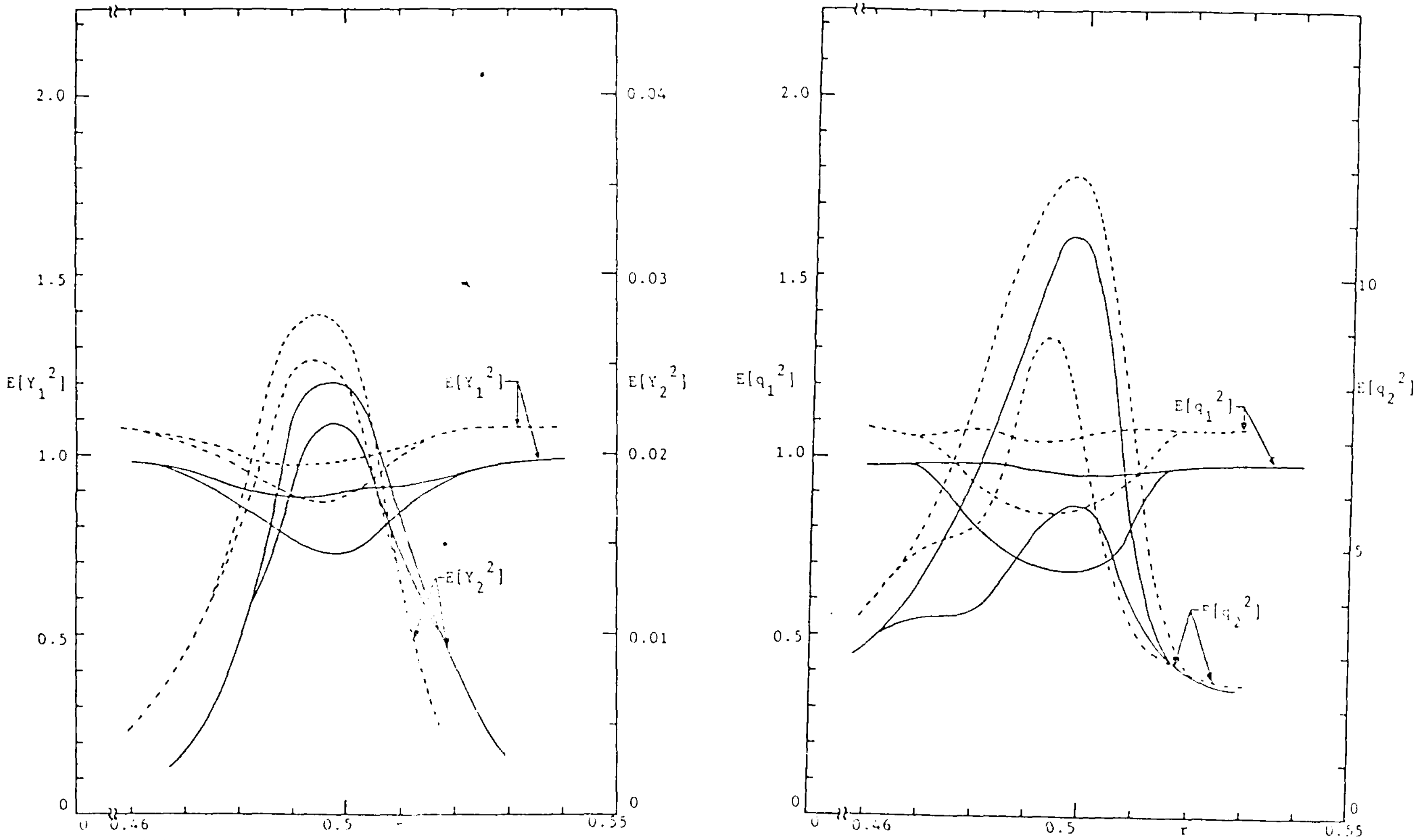


Fig. IV. 6 Time history of non-Gaussian response of system with constant damping ($D_{c_1} = D_{c_2} = 0$) and randomly varying stiffness ($D_{k_1} = D_{k_2} = 0.1$ D).

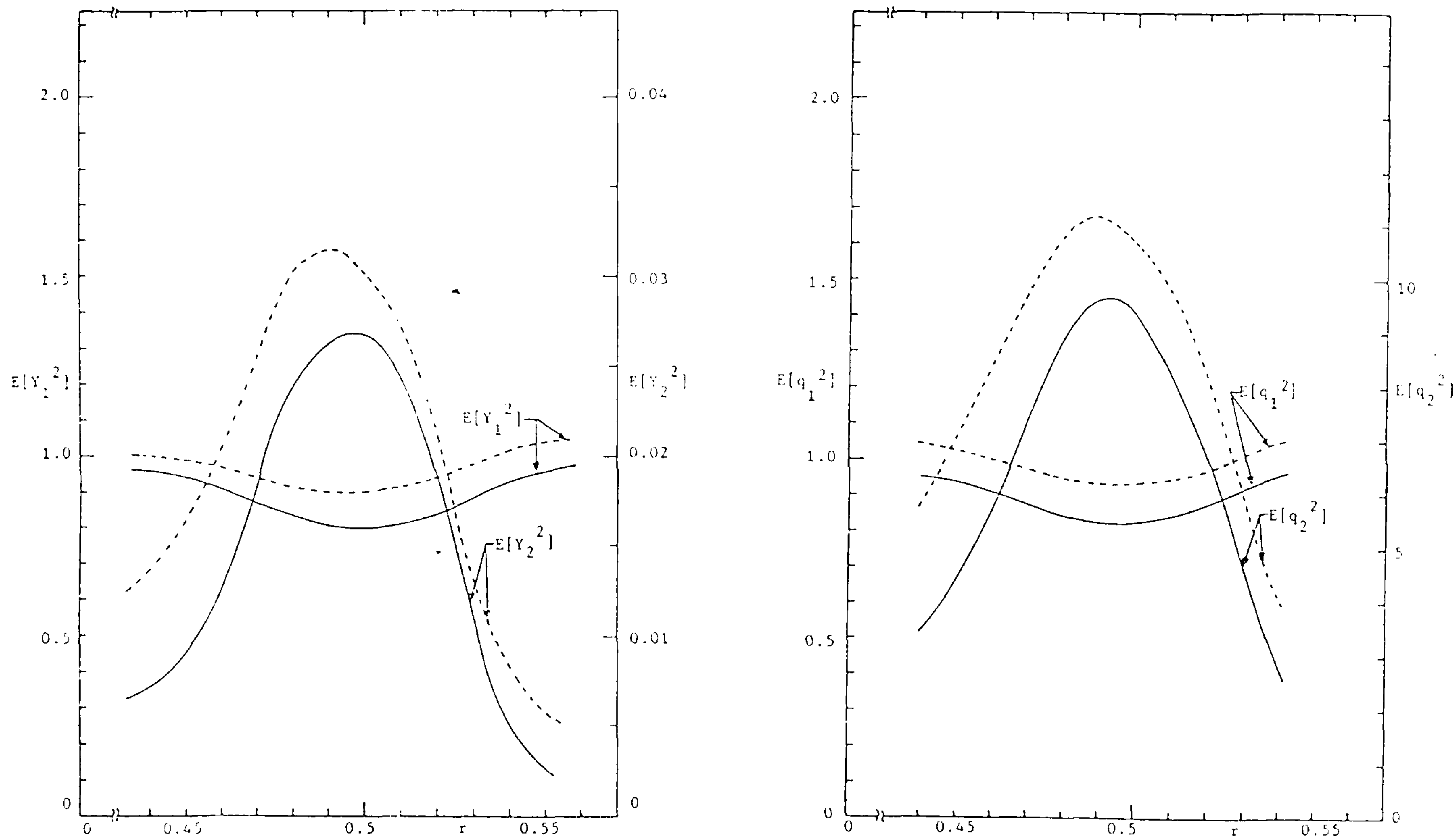


(a) Normal mode mean square response.

(b) Generalized mean square response.

Fig. IV.7 Gaussian closure solution for system with constant damping and randomly varying stiffness.

———— : constant damping and stiffness, - - - - - - : randomly varying stiffness ($D_{k_i} = 0.1\ D$, $D_{c_i} = 0$, $i = 1, 2$)



(a) Normal mode mean square response.

(b) Generalized mean square response.

Fig. IV. 3 Non-Gaussian closure solution for system with constant damping and randomly varying stiffness.

________: constant damping and stiffness, - - - - - -: randomly varying stiffness ($D_{k_i} = 0.1\ D$, $D_{c_i} = 0$, $i = 1, 2$)

IV.8 System Response with Damping and Stiffness Uncertainties

In this section the combined effect of the system damping and stiffness uncertainties on the system random response will be examined. Typical spectral densities for the uncertainties of these parameters are adopted, namely $D_{c_1} = D_{c_2} = D_{k_1} = D_{k_2} = 0.1\ D$, where $2D = 0.08$ is the spectral density of the base random motion.

The time history responses according to the Gaussian closure solution for internal tuning ratio $r = 0.5$ are shown in Fig. IV.9. Corresponding plots according to the non-Gaussian solution are shown in Fig. IV.10. The inclusion of the uncertainties of the system parameters bring the system response into a wider fluctuating limit cycle for the Gaussian closure solution or into a wider stationary limit cycle for the non-Gaussian closure solution. Furthermore, the transient response period with parameter uncertainties is relatively longer than the transient response period with constant system parameters. The mean squares of the response displacements during the steady state regime are plotted in Figs. IV.11 and IV.12 by dotted curves; the solid curves show the response of the system with constant parameters. The higher level of the response mean squares is due mainly to stiffness uncertainty rather than damping as inferred from the previous two sections.

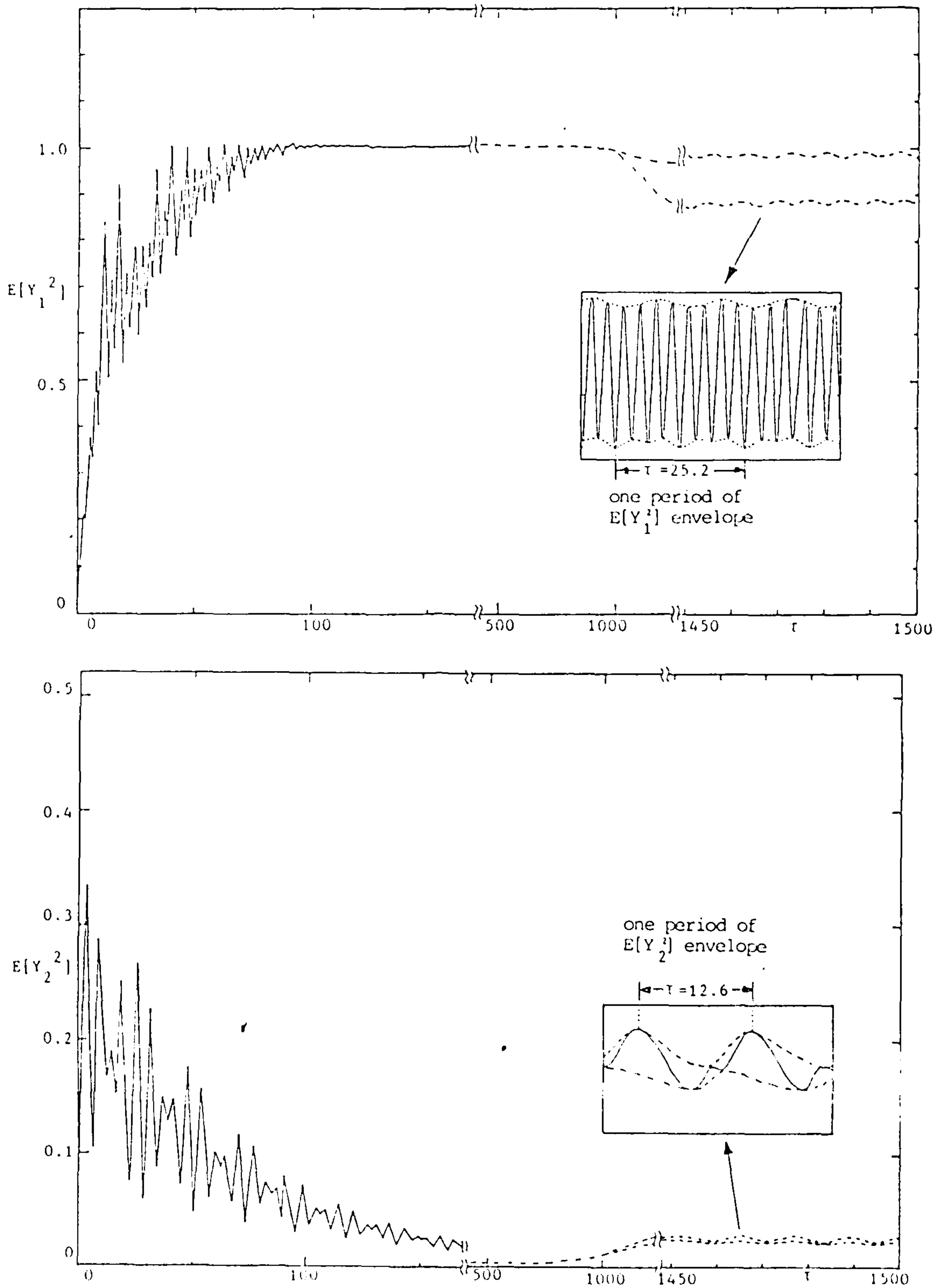
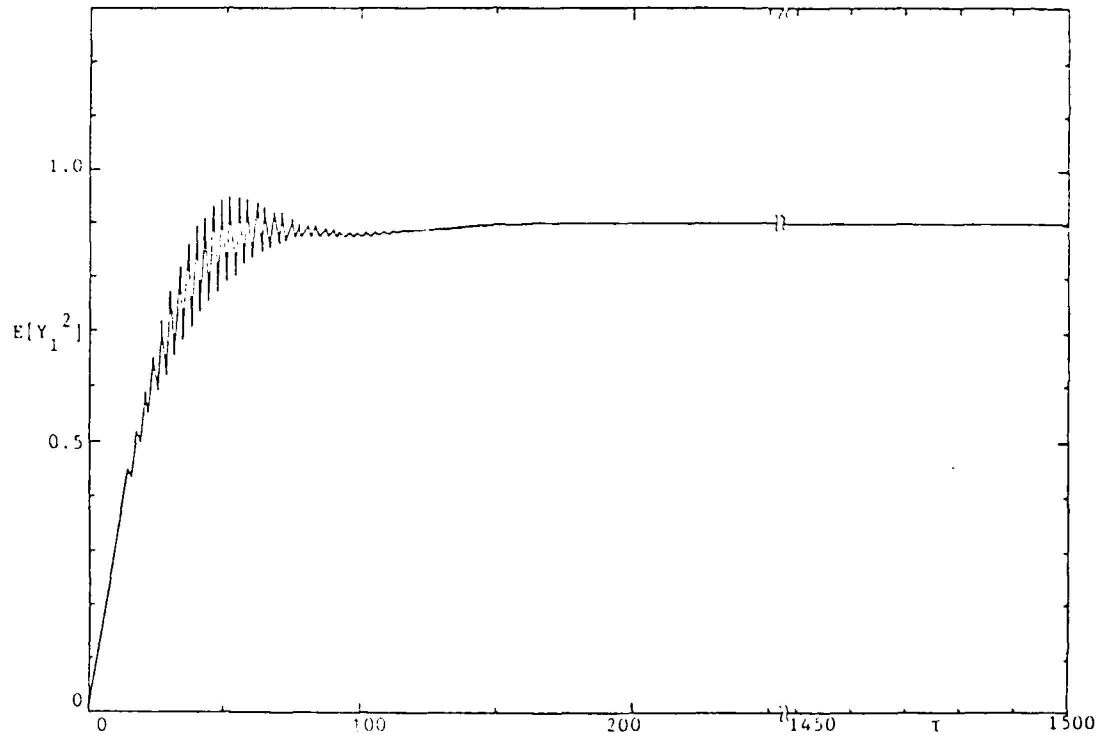


Fig. IV.9 Time history of Gaussian response of system with randomly varying damping ($D_{c_1} = D_{c_2} = 0.1$ D) and stiffness ($D_{k_1} = D_{k_2} = 0.1$ D).



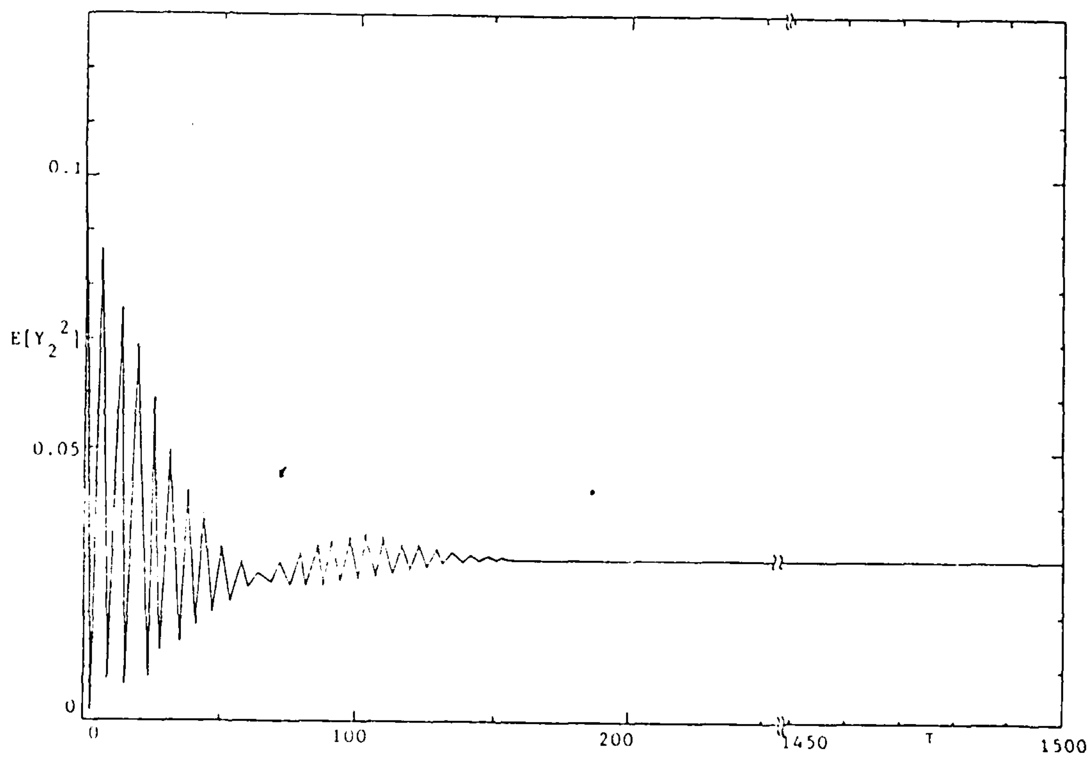


Fig. IV.10 Time history of non-Gaussian response of system with randomly varying damping ($D_{c_1} = D_{c_2} = 0.1\ D$) and stiffness ($D_{k_1} = D_{k_2} = 0.1\ D$).

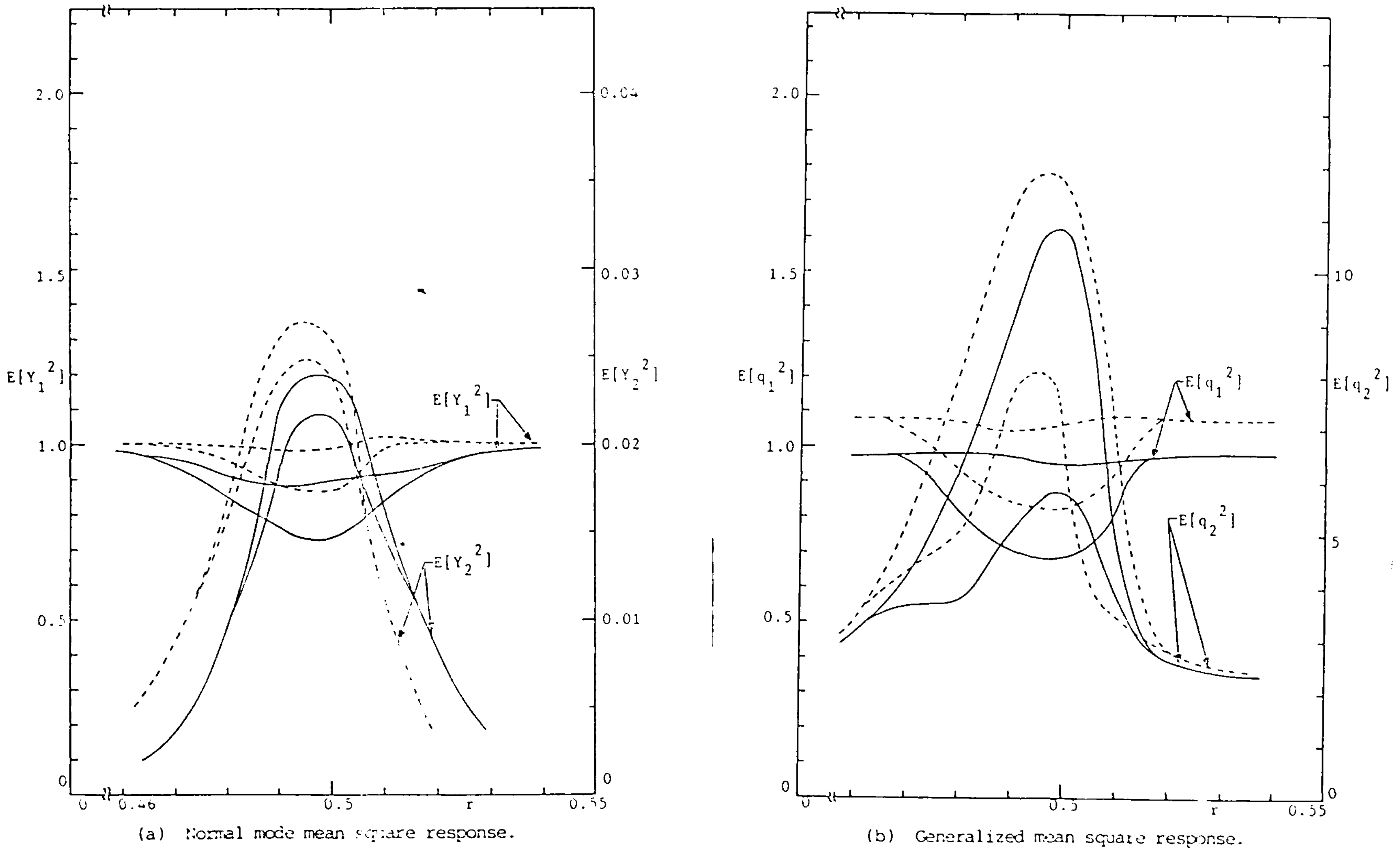
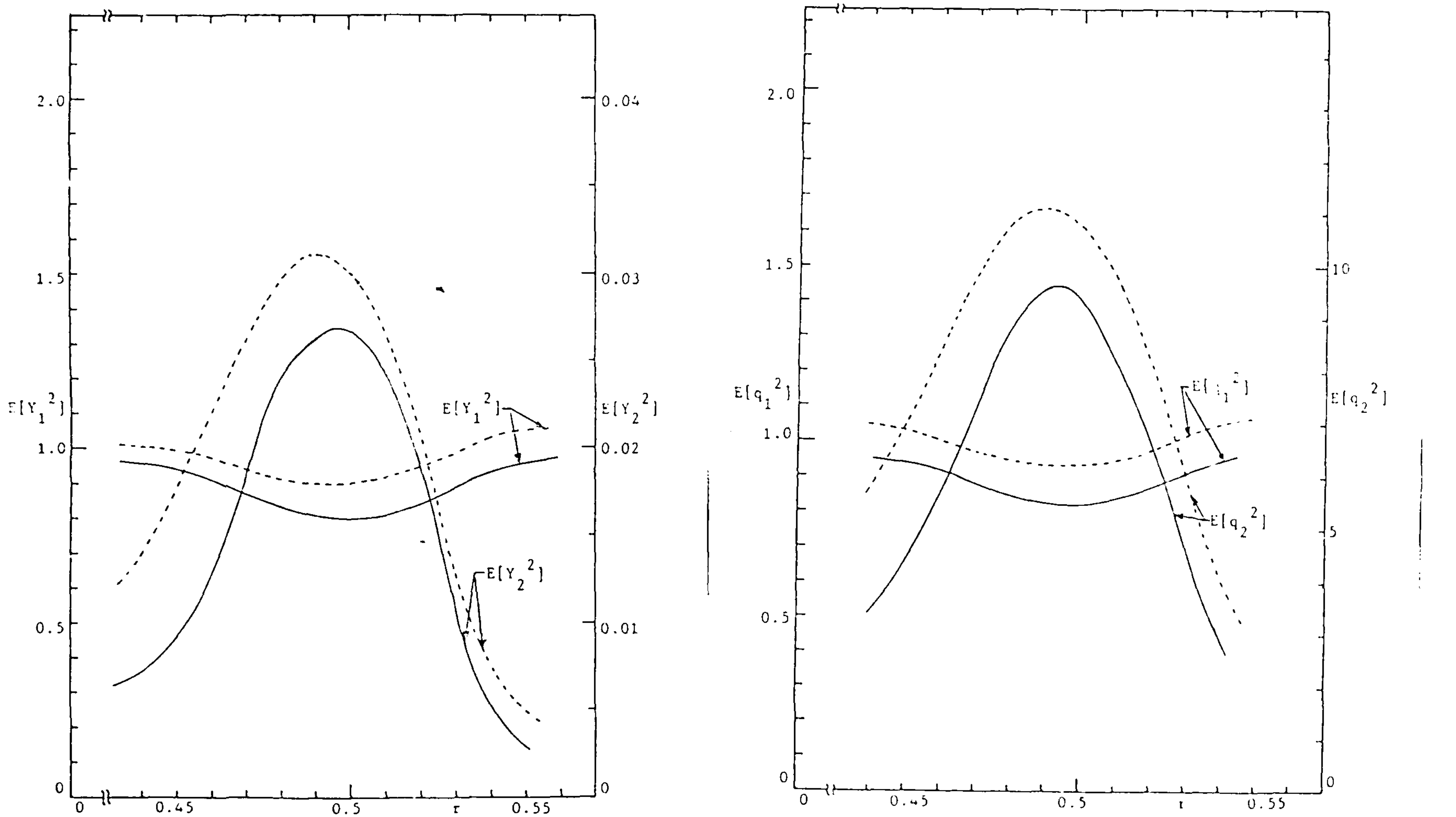


Fig. IV.11 Gaussian closure solution for system with randomly varying damping and stiffness.

________: constant damping and stiffness, - - - - - -: randomly varying damping and stiffness ($D_{c_i} = D_{k_i} = 0.1$ D, i = 1, 2)



(a) Normal mode mean square response.

(b) Generalized mean square response.

Fig. IV.12 Non-Gaussian closure solution for system with randomly varying damping and stiffness.

_______: constant damping and stiffness, - - - - - -: randomly varying damping and stiffness ($D_{c_i} = D_{k_i} = 0.1\ D$, $i = 1, 2$)

IV.9 Conclusions

The influence of uncertainties in system parameters such as damping and stiffness on the autoparametric response of an aeroelastic structure has been examined numerically. The damping influence upon the response characteristics is found to be very small while the stiffness uncertainty has a remarkable effect on both the level of the mean square response and the region of autoparametric interaction. The transient responses in the time domain are relatively higher than the steady state responses. This feature is very important in determining the actual stresses under random vibration. With stiffness uncertainty, the system response uncertainty becomes higher than when the system possesses constant stiffness.

CHAPTER V

CONCLUSIONS

The linear and non-linear random modal interactions of a two degree-of-freedom aeroelastic structure has been examined. The structure was subjected to a random wide band support motion. The equations of motion were derived by Lagrangian formulation and transformed into normal coordinates to eliminate the linear dynamic coupling. The random excitation appeared as parametric and non-homogeneous terms in the equations of motion.

The linear modal interaction was analyzed via the Fokker-Planck equation approach. The statistical moments of the system response were solved for various system parameters. The general trend of the linear solution showed that there is a suppression of one mode due to the presence of feedback forces from the other mode. Such interaction took place when the frequencies of the uncoupled beams are close to each other. Further analysis has shown that the parametric terms have a very small effect on the level of the response mean squares. However, these parametric excitations affect the stochastic stability of the system equilibrium configuration.

When the structure is tuned to the appropriate conditions of internal resonance, the non-linear coupling of normal modes becomes significant in predicting new response characteristics. The non-linear analysis is not a simple task and involves the problem of infinite hierarchy of differential equations of response moments. Two schemes have been used to close the response moment equations. The first, known as Gaussian closure, is based on the assumption that the response does not depart significantly from Gaussian distribution, while the second takes into consideration the

non-normality of the response and is known as the non-Gaussian closure method. The first scheme led to fourteen differential equations in the first and second order moments, while the second required sixty-nine differential equations in the first four orders of response moments. The two sets of differential equations were solved by numerical integration. Both solutions exhibited an energy exchange between the two modes in the neighborhood of the internal resonance condition $r = \frac{\omega_2}{\omega_1} = 0.5$. The Gaussian closure solution gave a quasi-stationary response in the form of fluctuations between two limits. However, the non-Gaussian solution resulted in a strict stationary response. The accuracy of the non-Gaussian solution required almost twenty times more computer time than the Gaussian solution.

In Chapter IV the damping and stiffness coefficients of the system were subjected to time random variations. It was found that the damping variation had very little influence on the response characteristics, while the stiffness variation showed a pronounced effect on the response mean squares for both solutions.

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APPENDIX A: COEFFICIENTS OF LINEAR PARAMETRIC EQUATIONS OF MOTION

$$E_1 = -\frac{1+m_{22}}{1+m_{22}(a+2\phi_1 b+\phi_1^2)}$$

$$E_2 = -\frac{m_{22}(1.5b+3\phi_1+1.8\phi_1^2/b)}{1+m_{22}(a+2\phi_1 b+\phi_1^2)}$$

$$E_3 = -\frac{m_{22}(1.5b+1.5(\phi_1+\phi_2)+1.8\phi_1\phi_2/b)}{1+m_{22}(a+2\phi_1 b+\phi_1^2)}$$

$$G_1 = -\frac{1+m_{22}}{1+m_{22}(a+2\phi_2 b+\phi_2^2)}$$

$$G_2 = -\frac{m_{22}(1.5b+1.5(\phi_1+\phi_2)+1.8\phi_1\phi_2/b)}{1+m_{22}(a+2\phi_2 b+\phi_2^2)}$$

$$G_3 = -\frac{m_{22}(1.5b+3\phi_2+1.8\phi_2^2/b)}{1+m_{22}(a+2\phi_2 b+\phi_2^2)}$$

APPENDIX B: COEFFICIENTS OF LINEAR PARAMETRIC SOLUTIONS

$$d_1 = D\cdot\varepsilon^2\cdot\left[\frac{m_{22}(1.5b+1.5(\phi_1+\phi_2)+1.8\phi_1\phi_2/b)}{1+m_{22}(a+2\phi_2 b+\phi_2^2)}\right]^2$$

$$d_2 = D\varepsilon^2\cdot\left[\frac{m_{22}(1.5b+3\phi_2+1.8\phi_2/b)}{1+m_{22}(a+2\phi_2 b+\phi_2^2)}\right]^2 - 2\zeta_2 r^3$$

$$d_3 = 2D\varepsilon^2\left[\frac{m_{22}(1.5b+1.5(\phi_1+\phi_2)+1.8\phi_1\phi_2/b)}{1+m_{22}(a+2\phi_2 b+\phi_2^2)}\right]\cdot$$

$$\left[\frac{m_{22}(1.5b+3\phi_2+1.8\phi_2^2/b)}{1+m_{22}(a+2\phi_2 b+\phi_2^2)}\right]$$

$$d_4 = -D\cdot\left[\frac{1+m_{22}}{1+m_{22}(a+2\phi_2 b+\phi_2^2)}\right]^2$$

$$e_1 = D\varepsilon^2\cdot\left[\frac{m_{22}(1.5b+3\phi_1+1.8\phi_1^2/b)}{1+m_{22})a+2\phi_1 b+\phi_1^2)}\right]^2 - 2\zeta_1$$

$$e_2 = D\varepsilon^2\cdot\left[\frac{m_{22}(1.5b+1.5(\phi_1+\phi_2)+1.8\phi_1\phi_2/b)}{1+m_{22}(a+2\phi_1 b+\phi_1^2)}\right]^2$$

$$e_3 = 2D\varepsilon^2 \left[\frac{m_{22}(1.5b+3\phi_1+1.8\phi_1^2/b)}{1+m_{22}(a+2\phi_1 b+\phi_1^2)}\right] \cdot \left[\frac{m_{22}(1.5b+1.5(\phi_1+\phi_2)+1.8\phi_1\phi_2/b)}{1+m_{22}(a+2\phi_1 b+\phi_1^2)}\right]$$

$$e_4 = -D \cdot \left[\frac{1+m_{22}}{1+m_{22}(a+2\phi_1 b+\phi_1^2)}\right]^2$$

$$f_1 = 2D\varepsilon^2 \cdot \left[\frac{m_{22}(1.5b+3\phi_1+1.8\phi_1^2/b)}{1+m_{22}(a+2\phi_1 b+\phi_1^2)}\right] \cdot \left[\frac{m_{22}(1.5b+1.5(\phi_1+\phi_2)+1.8\phi_1\phi_2/b)}{1+m_{22}(a+2\phi_2 b+\phi_2^2)}\right]$$

$$f_2 = 2D\varepsilon^2 \cdot \left[\frac{m_{22}(1.5b+1.5(\phi_1+\phi_2)+1.8\phi_1\phi_2/b)}{1+m_{22}(a+2\phi_1 b+\phi_1^2)}\right] \cdot \left[\frac{m_{22}(1.5b+3\phi_2+1.8\phi_2^2/b)}{1+m_{22}(a+2\phi_2 b+\phi_2^2)}\right]$$

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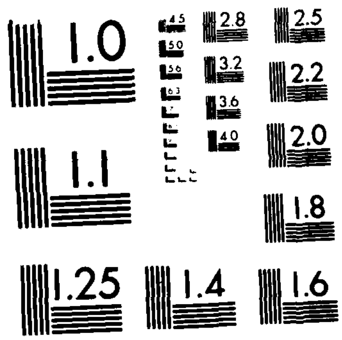

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$$f_3 = \left[r^2 - 1 + 4\zeta_1(\zeta_1 + r\zeta_2) \right] \frac{1-r^2}{2(\zeta_1 + r\zeta_2)}$$

$$+ 2 \left\{ D\varepsilon^2 \left\langle \left[\frac{m_{22}(1.5b + 3\phi_1 + 1.8\phi_1^2/b)}{1 + m_{22}(a + 2\phi_1 b + \phi_1)} \right] \cdot \left[\frac{m_{22}(1.5b + 3\phi_2 + 1.8\phi_2^2/b)}{1 + m_{22}(a + 2\phi_2 b + \phi_2^2)} \right] \right.\right.$$

$$\left.\left. + \frac{[m_{22}(1.5b + 1.5(\phi_1 + \phi_2) + 1.8\phi_1\phi_2/b)]^2}{[1 + m_{22}(a + 2\phi_1 b + \phi_1^2)] \cdot [1 + m_{22}(a + 2\phi_2 b + \phi_2^2)]} \right\rangle - \zeta_1 - r\zeta_2 \right\}$$

$$f_4 = -\,2D \cdot \frac{(1 + m_{22})^2}{[1 + m_{22}(a + 2\phi_1 b + \phi_1^2)] \cdot [1 + m_{22}(a + 2\phi_2 b + \phi_2^2)]}$$

APPENDIX C: COEFFICIENTS OF NON-LINEAR FUNCTIONS

$$a_1 = \frac{-(1 + m_2/m_1)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\frac{\ell_2}{\ell_1}\phi_1 + \phi_1^2\right)\frac{m_2}{m_1}\right)}$$

$$a_2 = \frac{-\frac{m_2}{m_1}\left(2.25\frac{\ell_2}{\ell_1} + 3\phi_1 + \frac{1.2}{\ell_2/\ell_1}\phi_1^2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\frac{\ell_2}{\ell_1}\phi_1 + \phi_1^2\right)\frac{m_2}{m_1}\right)}$$

$$a_3 = \frac{-\frac{m_2}{m_1}\left(2.25\frac{\ell_2}{\ell_1} + 1.5\,(\phi_1 + \phi_2) + \frac{1.2}{\ell_2/\ell_1}\phi_1\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\frac{\ell_2}{\ell_1}\phi_1 + \phi_1^2\right)\frac{m_2}{m_1}\right)}$$

$$a_4 = \frac{-\frac{m_2}{m_1}\left(0.9\frac{\ell_2}{\ell_1} + \frac{2.4}{\ell_2/\ell_1}\phi_1^2 + 3.6\phi_1\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\frac{\ell_2}{\ell_1}\phi_1 + \phi_1^2\right)\frac{m_2}{m_1}\right)}$$

$$a_5 = \frac{-\frac{m_2}{m_1}\left(0.9\frac{\ell_2}{\ell_1} + \frac{1.2}{\ell_2/\ell_1}\phi_1\,(\phi_1 + \phi_2) + 3.3\phi_1 + 0.3\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\frac{\ell_2}{\ell_1}\phi_1 + \phi_1^2\right)\frac{m_2}{m_1}\right)}$$

$$a_6 = \frac{-\frac{m_2}{m_1}\left(0.9\frac{\ell_2}{\ell_1} + \frac{2.4}{\ell_2/\ell_1}\phi_1\phi_2 + 0.6\phi_1 + 3\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\frac{\ell_2}{\ell_1}\phi_1 + \phi_1^2\right)\frac{m_2}{m_1}\right)}$$

$$a_7 = \frac{-\frac{m_2}{m_1}\left(0.9\,\frac{\ell_2}{\ell_1} + \frac{1.2}{\ell_2/\ell_1}\,\phi_2(\phi_1 + \phi_2) + 0.3\phi_1 + 3.3\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_1 + \phi_1^2\right)\frac{m_2}{m_1}\right)}$$

$$a_8 = \frac{-\frac{m_2}{m_1}\left(0.45\,\frac{\ell_2}{\ell_1} + \frac{1.2}{\ell_2/\ell_1}\,\phi_1^2 + 1.8\phi_1\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_1 + \phi_1^2\right)\frac{m_2}{m_1}\right)}$$

$$a_9 = \frac{-\frac{m_2}{m_1}\left(0.9\,\frac{\ell_2}{\ell_1} + \frac{2.4}{\ell_2/\ell_1}\,\phi_1\phi_2 + 0.6\phi_1 + 3\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_1 + \phi_1^2\right)\frac{m_2}{m_1}\right)}$$

$$a_{10} = \frac{-\frac{m_2}{m_1}\left(0.45\,\frac{\ell_2}{\ell_1} + \frac{1.2}{\ell_2/\ell_1}\,\phi_2^2 - 1.2\phi_1 + 3\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_1 + \phi_1^2\right)\frac{m_2}{m_1}\right)}$$

$$b_1 = \frac{-(1 + m_2/m_1)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_2 + \phi_2^2\right)\frac{m_2}{m_1}\right)}$$

$$b_2 = \frac{-\frac{m_2}{m_1}\left(2.25\,\frac{\ell_2}{\ell_1} + 1.5\,(\phi_1 + \phi_2) + \frac{1.2}{\ell_2/\ell_1}\,\phi_1\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_2 + \phi_2^2\right)\frac{m_2}{m_1}\right)}$$

$$b_3 = \frac{-\frac{m_2}{m_1}\left(2.25\,\frac{\ell_2}{\ell_1} + 3\phi_2 + \frac{1.2}{\ell_2/\ell_1}\,\phi_2^2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_2 + \phi_2^2\right)\frac{m_2}{m_1}\right)}$$

$$b_4 = \frac{-\frac{m_2}{m_1}\left(0.9\,\frac{\ell_2}{\ell_1} + \frac{1.2}{\ell_2/\ell_1}\,\phi_1\,(\phi_1 + \phi_2) + 3.3\phi_1 + 0.3\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_2 + \phi_2^2\right)\frac{m_2}{m_1}\right)}$$

$$b_5 = \frac{-\frac{m_2}{m_1}\left(0.9\,\frac{\ell_2}{\ell_1} + \frac{2.4}{\ell_2/\ell_1}\,\phi_1\phi_2 + 3\phi_1 + 0.6\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_2 + \phi_2^2\right)\frac{m_2}{m_1}\right)}$$

$$b_6 = \frac{-\frac{m_2}{m_1}\left(0.9\,\frac{\ell_2}{\ell_1} + \frac{1.2}{\ell_2/\ell_1}\,\phi_2\,(\phi_1 + \phi_2) + 0.3\phi_1 + 3.3\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_2 + \phi_2^2\right)\frac{m_2}{m_1}\right)}$$

$$b_7 = \frac{-\frac{m_2}{m_1}\left(0.9\,\frac{\ell_2}{\ell_1} + \frac{2.4}{\ell_2/\ell_1}\,\phi_2^2 + 3.6\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_2 + \phi_2^2\right)\frac{m_2}{m_1}\right)}$$

$$b_8 = \frac{-\frac{m_2}{m_1}\left(0.45\,\frac{\ell_2}{\ell_1} + \frac{1.2}{\ell_2/\ell_1}\,\phi_1^2 + 3\phi_1 - 1.2\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_2 + \phi_2^2\right)\frac{m_2}{m_1}\right)}$$

$$b_9 = \frac{-\frac{m_2}{m_1}\left(0.9\,\frac{\ell_2}{\ell_1} + \frac{2.4}{\ell_2/\ell_1}\,\phi_1\phi_2 + 3\phi_1 - 0.6\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_2 + \phi_2^2\right)\frac{m_2}{m_1}\right)}$$

$$b_{10} = \frac{-\frac{m_2}{m_1}\left(0.45\,\frac{\ell_2}{\ell_1} + \frac{1.2}{\ell_2/\ell_1}\,\phi_2^2 + 1.8\phi_2\right)}{\left(1 + \left(1 + 2.25\left(\frac{\ell_2}{\ell_1}\right)^2 + 3\,\frac{\ell_2}{\ell_1}\,\phi_2 + \phi_2^2\right)\frac{m_2}{m_1}\right)}$$

APPENDIX D: COEFFICIENTS OF NON-LINEAR MARKOV VECTOR EQUATIONS

$$A_1 = a_1$$

$$A_2 = a_1 a_4 + a_2 + a_5 b_1$$

$$A_3 = a_1 a_6 + a_3 + a_7 b_1$$

$$A_4 = a_2 a_4 + a_1 a_4^2 + a_5 b_1 b_5 + a_5 b_2$$

$$A_5 = a_3 a_4 + a_2 a_6 + 2 a_1 a_4 a_6 + a_5 b_1 b_7 + a_5 b_3 + a_7 b_1 b_5 + a_7 b_2$$

$$A_6 = a_1 a_6^2 + a_3 a_6 + a_7 b_1 b_7 + b_3 a_7$$

$$B_1 = b_1$$

$$B_2 = b_1 b_5 + b_2 + a_1 b_4$$

$$B_3 = b_1 b_7 + b_3 + b_6 a_1$$

$$B_4 = b_1 b_5^2 + b_2 b_5 + b_4 a_1 a_4 + a_2 b_4$$

$$B_5 = b_2 b_7 + b_3 b_5 + 2 b_1 b_5 b_7 + b_4 a_1 a_6 + b_4 a_3 + b_6 a_1 a_4 + b_6 a_2$$

$$B_6 = b_1 b_7^2 + b_3 b_7 + b_6 a_1 a_6 + b_6 a_3$$

4. PUBLICATIONS

1. Ibrahim, R. A. and Heo, H., "Autoparametric Vibration of Coupled Beams Under Random Support Motion," ASME Paper No. 85-DET-184, September 1985. Accepted for publication in the ASME Journal of Vibration, Acoustics, Stress and Reliability in Design.

2. Ibrahim, R. A. and Heo, H., "Stochastic Flutter of Non-linear Aeroelastic Structures," Abstract submitted for possible presentation (and publication in the AIAA Journal) at the AIAA/ASME/ASCE/AHS 27th Structures, Structural Dynamics and Materials Conference to be held in San Antonio, Texas, May 19-21, 1986.

5. PROFESSIONAL PERSONNEL

A. FACULTY

1. Raouf A. Ibrahim, Associate Professor, Department of Mechanical Engineering, Principal Investigator.

B. GRADUATE STUDENTS

1. Zhian Hedayati, M. S. Student in Mechanical Engineering. Thesis title: "Random Modal Interaction of a Non-linear Aeroelastic Structure," (completed). This student has accepted a scholarship from M.I.T. to complete his Ph.D. degree.

2. Hun Heo, Ph.D. Student in Mechanical Engineering. Thesis title: "Non-linear Stochastic Flutter of Aeroelastic Structural Systems," (in progress).

3. Douglas G. Sullivan, M. S. Student in Mechanical Engineering. Thesis title: "Experimental Investigation of Non-linear Random Flutter of Aeroelastic Structures," (in progress).

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